WORKED EXAMPLES

IN

ELECTRICAL ENGINEERING

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VOLUME I
DIRECT CURRENT

FOR FIRST, SECOND & THIRD YEAR DEGREE AND
SECOND, THIRD YEAR N. C. C. STUDENTS

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PREFACE

The book has been written to meet the demand of engineering students, who already have a sound knowledge of basic principles in electrical engineering and require guidance in their application.

A good variety of problems on 'Direct Current' have been solved in full detail in this volume. The methods adopted are not the only ones, nor necessarily the best; the main object has been to set the student thinking by providing him with the necessary analysis.

The book is not a text book and the theory given is sufficient for an understanding of the problems.

The authors earnestly hope that the book will prove useful to the students and enable them to tackle any problem in a systematic manner.

Although every care has been taken in checking the solutions, errors may still remain. Notification of any corrections, and suggestions will be gratefully accepted.

Patiala, 1960.

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CONTENTS

CHAPTER I

SPECIFIC RESISTANCE

1-1. Resistance R of a conductor is given by the expression :

$$R=\rho\frac{l}{a} \qquad \ldots \qquad \ldots\text{Eq. (1-1)}$$

where l=length of the conductor

a=area of cross-section of the conductor

& ρ=specific resistance or resistivity for the material of the conductor.

1-2. Specific Resistance (or resistivity) is defined as the resistance of a conductor of unit length and unit cross-sectional area. If 'l' and 'a' are in inch units, ρ is expressed in *ohms per inch cube*, if 'l' and 'a' are in centimeter units, ρ is expressed in *ohms per* cm. cube.

Sometimes ρ is stated in *ohms per circular mil-foot*. This is the resistance of a wire 1 foot long, and 1 mil in diameter, where

$$\text{one mil} = \frac{1}{1000} \text{ inch.}$$

The area of a circle whose diameter is one mil, is one circular mil.

i.e., $\text{one circular mil} = \frac{1}{1000}\times\frac{1}{1000}\times\frac{\pi}{4}$ sq. inches

$$= \frac{\pi}{4}\times 10^{-6} \text{ Sq. inches.}$$

so that if diameter of a conductor is 3 mils, its cross sectional

area $= \frac{3}{1000}\times\frac{3}{1000}\times\frac{\pi}{4}$ sq. inches.

$$=9\times\frac{\pi}{4}\times 10^{-6} \text{ sq. inch.}$$

$$=9 \text{ circular mils}$$

$$(\because \ 1 \text{ circular mil} = \frac{\pi}{4}\times 10^{-6} \text{ sq. inch}).$$

Hence the area of a circle, d mils in diameter, is d^2 circular mils.

1-3. Specific resistance for copper $= \cdot 66 \times 10^{-6}$ ohm per inch cube or, $1 \cdot 66 \times 10^{-6}$ ohm per cm. cube

Specific resistance for Aluminium

$$= 1 \cdot 1 \times 10^{-6} \text{ ohm per inch cube.}$$

Example 1. An aluminium conductor is 100 yds. long and has a resistance of 2 ohms. Find its area of cross-section and the diameter given that specific resistance per inch cube for aluminium $= 1 \cdot 04$ microhms.

$$R = \rho \frac{l}{a}$$

$$2 = 1 \cdot 04 \times 10^{-6} \times \frac{100 \times 36}{a}$$

$$a = 1 \cdot 04 \times 10^{-6} \times \frac{100 \times 36}{2} = \cdot 00187 \text{ sq. in.}$$

If d is the diameter of the conductor

then $\frac{\pi d^2}{4} = \cdot 00187$

$$d = \sqrt{\cdot 00187 \times \frac{4}{\pi}} = \cdot 04879''. \quad \textit{Ans.}$$

Example 2. Find the resistance of a copper bus-bar 30 ft. long and $3'' \times \frac{1}{2}''$ in. cross-section. Specific resistance of copper is $\cdot 66 \times 10^{-6}$ ohm per in. cube.

$$R = \cdot 66 \times 10^{-6} \times \frac{30 \times 12}{3 \times \frac{1}{2}} = 158 \cdot 4 \times 10^{-6} \text{ ohm.}$$

$= 158 \cdot 4$ microhms. *Ans.*

Example 3. The resistance of 1000 yds. of $7/\cdot 029''$ copper conductor is $5 \cdot 281$ ohms. Find the approximate resistance per mile at the same temperature of a $3/\cdot 036''$ conductor.

$$\frac{R_2}{R_1} = \frac{l_2}{l_1} \times \frac{a_1}{a_2}$$

R_2 = Resistance to be found

$R_1 = 5{\cdot}281$

$l_2 = 1760$ yds.

$l_1 = 1000$ yds.

$a_2 = 3 \times \frac{\pi}{4} \times {\cdot}036^2$

$a_1 = 7 \times \frac{\pi}{4} \times {\cdot}029^2$

$$R_2 = 5{\cdot}281 \times \frac{1760}{1000} \times \frac{7 \times {\cdot}029^2}{3 \times {\cdot}036^2}$$

$$= 5{\cdot}281 \times \frac{1760}{1000} \times \frac{7 \times {\cdot}00841}{3 \times {\cdot}013} = 14{\cdot}11 \text{ ohm. } \textit{Ans.}$$

Example 4. The resistance of one mile of copper wire ·134″ diameter is 3·12 ohms. Calculate the resistance of a quarter mile of german silver wire ·065″ diameter given that the specific resistance of german silver is 13 times that of copper.

$$\frac{R_2}{R_1} = \frac{13\rho}{\rho} \times \frac{l_2}{a_2} \times \frac{a_1}{l_1}$$

$R_1 = 3{\cdot}12$

$l_2 = \frac{1}{4}$ mile

$l_1 =$ one mile

$a_2 = {\cdot}065^2 \times \frac{\pi}{4}$

$a_1 = {\cdot}134^2 \times \frac{\pi}{4}$

$$R_2 = 3{\cdot}12 \times 13 \times \frac{1}{4} \times \frac{{\cdot}134^2}{{\cdot}065^2} = 43{\cdot}2 \text{ ohms. } \textit{Ans.}$$

Example 5. Find the resistance of a conductor 1000 ft. long and 75 mils in diameter given resistance per mil foot $= 9{\cdot}8$ ohm.

If ρ is the specific resistance per inch cube

$$9{\cdot}8 = \rho \times \frac{12}{\frac{\pi}{4} \times \frac{1}{1000} \times \frac{1}{1000}}$$

or $$\rho = \frac{9{\cdot}8 \times \frac{\pi}{4} \times 10^{-6}}{12}$$

$$\text{Resistance} = \frac{9{\cdot}8 \times \frac{\pi}{4} \times 10^{-6}}{12} \times \frac{1000 \times 12}{75 \times 75 \times \frac{\pi}{4} \times 10^{-6}}$$

$$= \frac{9{\cdot}8 \times 1000}{75 \times 75} = 1{\cdot}742 \text{ ohms.} \quad \textit{Ans.}$$

Obviously

$$\text{Resistance} = \frac{\text{Resistance per mil foot} \times \text{length in ft.}}{\text{Area in circular mils.}}$$

or $$= \frac{\text{Res. per mil yard} \times \text{length in yds.}}{\text{Area in circular mils.}}$$

Example 6. Determine the specific resistance per inch cube and per cm. cube of a wire 5 miles long and 10 mils diameter which has a resistance of 200 ohms.

$$200 = \rho \times \frac{5 \times 5280 \times 12}{\frac{\pi}{4} \times \frac{100}{10^6}}$$ where ρ=specific Res. per inch cube.

$$\text{or } \rho = 200 \times \frac{\pi}{4} \times \frac{1}{10^4} \times \frac{1}{5 \times 5280 \times 12}$$
$$= {\cdot}0496 \times 10^{-6} \text{ ohm per inch cube.}$$

Specific resistance per cm. cube $= {\cdot}0496 \times 10^{-6} \times 2{\cdot}54$
$= {\cdot}1259 \times 10^{-6}$ ohm per cm. cube. *Ans.*

Example 7. Find the resistance of 125 yds. of wire 75 mils in diameter given that the resistance of one mile of wire of the same material 24 mils in diameter is ·01 ohm.

$$\text{Res. per mil yd.} = \frac{{\cdot}01 \times 24 \times 24}{1760}$$

$$\text{Resistance required} = \frac{{\cdot}01 \times 24 \times 24}{1760} \times \frac{125}{75 \times 75} = \frac{{\cdot}08}{1100}$$
$$= 72{\cdot}7 \times 10^{-6} \text{ ohm.} \quad \textit{Ans.}$$

Example 8. A nichrome heater spiral rated at 1000 watts, 230 volts consists of 180 turns, each having a mean diameter of 0·6 inch, the diameter of the wire being 0·018 inch. Calculate the resistivity (at the operating temperature) of the material.

$$\frac{E^2}{R}=1000$$

or $$R=\frac{230\times230}{1000}=52{\cdot}9 \text{ ohms.}$$

Length of the heater wire $=180\times\pi\times{\cdot}6$

Cross-sectional area of the wire $=\frac{\pi}{4}\times{\cdot}018^2$

$\therefore$ $$52{\cdot}9=\frac{\rho\times180\times\pi\times{\cdot}6}{\frac{\pi}{4}\times{\cdot}018\times{\cdot}018}$$

or $$\rho=\frac{\pi}{4}\times\frac{{\cdot}018\times{\cdot}018\times52{\cdot}9}{180\times\pi\times{\cdot}6}$$

$=39{\cdot}7\times10^{-6}$ ohm per inch cube. *Ans.*

Example 9. Two wires one of aluminium and the other of copper have the same diameter and each weighs ½ lb. The copper wire is 20 ft. long. Calculate the length of the aluminium wire and the resistance of each.

Take	ρ for copper	$=1{\cdot}7$ microhms per cm. cube	
	ρ for Al.	$=2{\cdot}6$	,, ,, ,,
Sp. gravity of copper		$=8{\cdot}89$	
Sp. gravity of Al.		$=2{\cdot}7.$	

Weight=Volume × sp. gravity

or $$a_1 l_1 s_1=a_2 l_2 s_2$$

a_1 and a_2 are equal

$\therefore$ $$l_1 s_1=l_2 s_2$$

Length of aluminium conductor

$$=\frac{20\times8{\cdot}89}{2{\cdot}7}=65{\cdot}8 \text{ ft.}$$

$$R = \rho\frac{l}{a} = \rho \times \frac{l}{\dfrac{\text{weight}}{l \times \text{sp. gravity}}} = \frac{\rho l^2 s}{w}$$

$$\text{Wt. of each wire} = \frac{453 \cdot 6}{4} = 113 \cdot 4 \text{ gms.}$$

$$\text{Res. of copper wire} = \frac{1 \cdot 7 \times 10^{-6} \times (20 \times 12 \times 2 \cdot 54)^2 \times 8 \cdot 89}{113 \cdot 4}$$

$$= \cdot 0495 \text{ ohm}$$

$$\text{Res. Al. of conductor} = \frac{2 \cdot 6 \times 10^{-6} \times (65 \cdot 8 \times 12 \times 2 \cdot 54)^2 \times 2 \cdot 7}{113 \cdot 4}$$

$$= \cdot 247 \text{ ohm} \quad \textit{Ans.}$$

Example 10. The resistance of a mile of iron wire weighing 100 lbs is 54 ohms. Find the resistance of an iron rod 2 yds. long weighing one lb.

$$\text{Wt. of wire per yard} = \frac{100}{1760} \text{ lb}$$

$$\text{Wt. of rod per yard} = \frac{1}{2} \text{ lb}$$

$$\frac{\text{Sectional area of rod}}{\text{Sectional area of wire}} = \frac{A_2}{A_1} = \frac{1}{2} \times \frac{1760}{100}$$

$$\frac{R_2}{R_1} = \frac{l_2}{l_1} \times \frac{A_1}{A_2}$$

$$R_2 = 54 \times \frac{2}{1760} \times \frac{200}{1760}$$

$$= 6 \cdot 96 \times 10^{-3} \text{ ohms. } \textit{Ans.}$$

Example 11. A dynamo supplies current to a set of 200 lamps which are grouped in parallel at a distance of $\frac{1}{2}$ mile from the dynamo. The leads have a cross-section of ·05 sq. in. If each lamp takes ·3 amp. and a P.D. of 220 V is maintained between the terminals of the group, what must be the P.D. at the dynamo terminal.

$$\text{Current taken by 200 lamps} = 200 \times \cdot 3 = 60 \text{ amps.}$$

$$\text{Resistance of the two leads} = \frac{\cdot 66 \times 10^{-6} \times 1760 \times 36}{\cdot 05}$$

$$= \cdot 835 \text{ ohm.}$$

$$\text{Voltage drop in leads} = \cdot 835 \times 60 = 50 \text{ V.}$$

$$\text{P.D. at dynamo terminals} = 220 + 50 = 270 \text{ volts. } \textit{Ans.}$$

Example 12. Current is required along a street 1000 feet long at the rate of one amp. per 10 ft. of frontage. Find the sectional area of the conductor so that the difference of pressure between any two lamps shall not exceed 5 volts in the following two cases :—

(*a*) Current being supplied at one end.

(*b*) ,, ,, ,, in the middle of the street.

Take ρ for copper $= 2/3 \times 10^{-6}$ ohm. per inch cubes.

(*a*) Total current $= \dfrac{1000}{10} = 100$ amps.

Mean value of current in the lines $=$ 50A

If R is the line resistance we have

$$50R = 5$$

$$R = \frac{1}{10} \text{ ohm.}, \quad l = 2000 \text{ ft.}$$

$$\frac{1}{10} = \frac{2}{3} \times 10^{-6} \times \frac{2000 \times 12}{a}$$

$$a = \frac{2}{3} \times 10^{-6} \times 2000 \times 12 \times 10$$

$$= \frac{16}{100} = \cdot 16 \text{ sq. in. } \textit{Ans.}$$

(*b*) Current is supplied in the middle of the street.

Current on each half side $=$ 50 amps.

Mean value of current $=$ 25 A

If R is resistance of line in half the street.

$$25R = 5$$

$$R = \frac{1}{5} \text{ ohm.}$$

$$l = 1000 \text{ ft.}$$

$$\frac{1}{5} = \frac{2}{3} \times 10^{-6} \times \frac{1000 \times 12}{a}$$

$$a = \frac{2}{3} \times 10^{-6} \times 1000 \times 12 \times 5$$

$$= \cdot 04 \text{ sq. in. } \textit{Ans.}$$

CHAPTER II

RESISTANCE TEMPERATURE COEFFICIENT

2-1. (*a*) Pure metals increase in resistance with rise of temperature.

(*b*) Alloys—Most alloys increase very slightly in resistance with rise of temperature. The resistance alloys used in electrical work, have a practically constant resistance at all temperatures, and therefore they are very suitable for making standard resistances.

(*c*) Carbon, insulators and electrolytes decrease in resistance with rise of temperature.

The resistance temperature graph of pure metals and alloys is practically a straight line.

2-2. A conductor having one ohm. resistance at 0°C increases in resistance by an amount α for each degree centigrade rise and becomes $1+\alpha t$ at t°C,

$$\therefore \quad R_t = R_0(1+\alpha t) \qquad \text{Eq. (2-1)}$$

where R_t = Resistance at a temp. t°C

R_0 = Resistance at a temp. 0°C.

α, is called **Resistance temp. coefficient** of the material and is defined as the increase in the resistance of the conductor having an original resistance of one ohm. at 0°C when the temperature rises by 1 °C

Values of α for	Copper	=0·00428
,, ,,	Manganin	=0·00002
,, ,,	Platinum silver	=0·00027
,, ,,	German silver	=0·00044

$$R_2 = R_1[1+\alpha_1(t_2-t_1)] \qquad \text{Eq. (2-2)}$$

where R_1 is the resistance at a temperature t_1, R_2 the resistance at a higher temperature t_2 and α_1 the res. temp. coefficient at temp. t_1.

If α_1 is the res. temp. coefficient of a conductor expressed as a fraction at temp. t_1°C, then the coefficient at temp. t_2°C is given by α_2

where
$$\alpha_2=\frac{1}{\frac{1}{\alpha_1}+(t_2-t_1)} \qquad \text{Eq. (2-3)}$$

Example 1. A length of nickel wire has a resistance of 50 ohms. at 10°C. The resistance of this wire at 60°C is found to be 60 ohms. Find the Res. temp. coefficient of nickel at 0°C.

Let the coefficient at 0°C be α

Then
$$R_{10}=50=R_0(1+\alpha\times 10)$$
$$R_{60}=60=R_0(1+\alpha\times 60)$$
$$\frac{50}{60}=\frac{5}{6}=\frac{1+10\alpha}{1+60\alpha}$$

or
$$5+300\alpha=6+60\alpha$$
$$240\alpha=1$$
$$\alpha=\cdot 00416 \text{ ohm. per degree C. } \textit{Ans.}$$

Example 2. The resistance of a copper cable was found to be 216 ohms. at a temp. of 20°C. The cable was then cooled and the resistance found to be 200 ohms. Find its final temperature. Take $\alpha=\cdot 004$.

Let the final temp. be t°C
$$R_{20}=R_0(1+\alpha\times 20)=216$$
$$R_t=R_0(1+\alpha\times t)=200$$
$$\frac{1+20\alpha}{1+\alpha t}=\frac{216}{200}=\frac{27}{25}$$
$$25+500\alpha=27+27\alpha t$$
$$25+2=27+27\alpha t$$

or
$$27\alpha t=0$$
$$t=0$$
$$=0\text{°C. } \textit{Ans.}$$

Example 3. Find the percentage change in the resistance of a motor armature from the initial room temperature of 25°C to the normal working temperature of 65°C.

$$R_2 = R_0(1+\alpha\times 65)$$
$$R_1 = R_0(1+\alpha\times 25)$$
$$\frac{R_2-R_1}{R_1} = \frac{1+65\alpha-1-25\alpha}{1+25\alpha} = \frac{40\alpha}{1+25\alpha}$$

Taking value of $\alpha = \cdot 004$

$$\frac{40\alpha}{1+25\alpha} = \frac{\cdot 16}{1\cdot 1}$$

$$\%\text{age change} = \frac{\cdot 16}{1\cdot 1}\times 100 = \frac{16}{1\cdot 1}$$
$$= 14\cdot 5\%. \quad \textit{Ans.}$$

Example 4. During a test on a shunt motor it was found that the resistance of the field coils is 57·5 ohms. If the resistance before beginning the test was 52·5 ohms. at a temp. of 15°C, find the mean temperature of the coil. Take $\alpha = \cdot 0042$ at 0°C.

$$\frac{R_2}{R_1} = \frac{1+\alpha t_2}{1+\alpha t_1}$$

or

$$\frac{R_2-R_1}{R_1} = \frac{\alpha(t_2-t_1)}{1+\alpha t_1}$$
$$t_2 - t_1 = \frac{R_2-R_1}{R_1}\times\frac{1+\alpha t_1}{\alpha}$$
$$= \frac{57\cdot 5-52\cdot 5}{52\cdot 5}\times\frac{1+\cdot 0042\times 15}{\cdot 0042} = 24°$$

Final temperature $= 24+15 = 39°$C. *Ans.*

Example 5. A coil has 1000 turns of copper wire with a cross-section of 1288 circular mils and a length of mean turn of 15 inches. Find:

(*a*) Res. of the coil at 0°C, given resistance per circular mil foot at 0°C = 9·7 ohms.

(*b*) Resistance at 25°C. Take $\alpha = \cdot 0042$.

(*c*) Current flowing through the coil at 25°C and 110 volts.

(*d*) After the current has passed for some time, its value is found to be 9 amps. Find the average temperature of the coil.

(a) Resistance at 0°C $= \dfrac{9.7 \times 1000 \times 15}{12 \times 1288} = 9.4$ ohms.

(b) Resistance at 25°C $= 9.4(1 + .0042 \times 25) = 10.3$ ohms.

(c) Current flowing $= \dfrac{110}{10.3} = 10.7$ amps.

(d) The resistance of the heated coil

$$= \frac{110}{9} = 12.2 \text{ ohms.}$$

$$12.2 = 9.4(1 + .0042t)$$

$$t = 70.9^\circ\text{C. } \textit{Ans.}$$

Example 6. (a) Show that if α_1 be the Resistance temperature coefficient of a conductor at t_1°C expressed as a fraction, the coefficient at temp. t_2°C is given by

$$\alpha_2 = \frac{1}{\dfrac{1}{\alpha_1} + (t_2 - t_1)}.$$

(b) A copper wire has a sp. resistance of 1.6×10^{-6} ohm. per cm. cube at 0°C and a resistance temp. coefficient of $\dfrac{1}{254.5}$ at 20°C. Find the temp. coefficient and specific resistance at 60°C.

$$\frac{R_2}{R_1} = \frac{1 + \alpha t_2}{1 + \alpha t_1}$$

or $$R_2 = R_1 \times \frac{1 + \alpha t_2}{1 + \alpha t_1}$$

also $$R_2 = R_1[1 + \alpha_1(t_2 - t_1)]$$

$$R_1[1 + \alpha_1(t_2 - t_1)] = R_1 \times \frac{1 + \alpha t_2}{1 + \alpha t_1}$$

$$\alpha_1(t_2 - t_1) = \frac{1 + \alpha t_2 - 1 - \alpha t_1}{1 + \alpha t_1} = \frac{\alpha(t_2 - t_1)}{1 + \alpha t_1}$$

$$\alpha_1 = \frac{\alpha}{1 + \alpha t_1} = \frac{1}{\dfrac{1}{\alpha} + t_1} \qquad \text{...(1)}$$

Similarly $$\alpha_2=\frac{1}{\frac{1}{\alpha}+t_2} \qquad \ldots(2)$$

From (1) $$\frac{1}{\alpha_1}=\frac{1}{\alpha}+t_1$$

or $$\frac{1}{\alpha}=\frac{1}{\alpha_1}-t_1$$

Putting this value of $\frac{1}{\alpha}$ in (2)

$$\alpha_2=\frac{1}{\frac{1}{\alpha_1}-t_1+t_2}=\frac{1}{\frac{1}{\alpha_1}+(t_2-t_1)}$$

(b) Temp. coefficient $$\alpha_{60}=\frac{1}{\frac{1}{\alpha_{20}}+(60-20)}$$

$$=\frac{1}{254\cdot5+40}=\frac{1}{294\cdot5}$$

Specific res. at 60°C

$$\alpha_{20}=\frac{1}{\frac{1}{\alpha_0}+20}$$

or $$\frac{1}{\alpha_{20}}=\frac{1}{\alpha_0}+20$$

$$254\cdot5=\frac{1}{\alpha_0}+20$$

or $$\frac{1}{\alpha_0}=234\cdot5$$

$$\alpha=\frac{1}{234\cdot5}$$

$$\text{Sp. Res. at } 60°\text{C}=1\cdot6\times10^{-6}\left(1+\frac{1}{234\cdot5}\times60\right)$$

$$=1\cdot6\times10^{-6}\times\frac{294\cdot5}{234\cdot5}$$

$$=2\cdot01\times10^{-6} \text{ ohm. per cm. cube.}$$

Example 7. Find the current flowing at the instant of switching on a 40 watt, 240 volt lamp given that the incandescent filament temperature is 2015°C and the resistance temp coefficient at 15°C is ·005.

$$\text{Filament res. at } 2015°\text{C} = \frac{240 \times 240}{40} = 1440 \text{ ohms}$$

$$R_2 = R_1(1 + \alpha_1 t)$$

where R_1 = Res. at 15°C

α_1 = Res. temp coefficient at 15°C

$$\therefore \quad 1440 = R_1(1 + \cdot005 \times 2000)$$

$$R_1 = 131 \text{ ohm.}$$

At the time of switching on the element is cold and has resistance of 131 ohms.

$$\text{Current taken} = \frac{240}{131} = 1{\cdot}83 \text{ Amps. } Ans$$

CHAPTER III

INSULATION RESISTANCE

3·1. *The* **insulation resistance** *of a cable is*

$$\frac{\rho}{2\pi l} \log_e \frac{r_2}{r_1} = \cdot 366 \frac{\rho}{l} \log_{10} \frac{r_2}{r_1} \qquad \text{Eq. (3-1)}$$

where ρ=Specific resistance of the insulating material

l=Length of the cable

r_1=Radius of the conductor

r_2=Radius of the outer surface of insulation.

From the above formula it is obvious that the insulation resistance of a cable varies inversely as its length.

3·2. Effect of temperature rise on the insulation :— The insulation resistance falls very rapidly with temperature rise, obeying the formula :—

$$\text{Log } R_t = \text{Log } R_s - K(t-s) \qquad \text{Eq. (3-2)}$$

where R_s=Insulation resistance at a standard temp. s°

R_t=Insulation resistance at a temp. t°.

K is a constant for the insulating material.

If t' is the temperature rise to halve the insulation resistance of a cable we have :—

$$\text{Log } 1 = \text{Log } 2 - Kt'$$

$$\text{or } Kt' = \text{Log } 2$$

$$\therefore \quad K = \frac{\text{Log } 2}{t'}$$

$\therefore$ Substituting the value of K in Eq. (3-2)

$$\text{Log } R_t = \text{Log } R_s - \frac{\text{Log } 2}{t'}(t-s) \qquad \text{Eq. (3-3)}$$

Example 1. A uniform cable 110 yards long when tested gave the conductor resistance ·22 ohm and insulation resistance 5000 megohms. Find the values per mile.

Conductor resistance varies directly as the length.

Conductor res. per mile $= \cdot 22 \times \frac{1760}{110} = 3 \cdot 52$ ohms. *Ans.*

Insulation resistance varies inversely as the length.

Insulation res. per mile $= 5000 \times \frac{110}{1760}$

$= 312 \cdot 5$ megohms. *Ans.*

Example 2. The total insulation resistance of a telegraph line between stations A and C is 5000 ohms. The insulation resistance of the line between A and an intermediate station B is 7000 ohms. What is the insulation resistance of the section BC and its insulation resistance per mile if this section is 5 miles long.

If R is the insulation resistance of section BC

$$\frac{1}{5000} = \frac{1}{7000} + \frac{1}{R}$$

$$\frac{1}{R} = \frac{1}{5000} - \frac{1}{7000} = \frac{2}{35000}$$

or $R = 17500$ ohms.

Insulation resistance per mile $= 17500 \times 5 = 87500$ ohms. *Ans.*

Example 3. An insulated cable is one mile long having a core of 2·5 mm. diameter. The thickness of the insulating material is 1·25 mm. and its resistivity $4 \cdot 5 \times 10^{14}$ ohms per cm. cube. Find the insulation resistance of the cable.

$$\text{Insulation Res.} = \cdot 366 \times \frac{\rho}{l} \cdot \log_{10} \frac{r_2}{r_1}$$

$$l = 1760 \times 36 \times 2 \cdot 54 \text{ cm.}$$

$$r_2 = 2 \cdot 5 \text{ mm.}$$

$$r_1 = 1 \cdot 25 \text{ mm.}$$

$$\text{Insulation res.} = \cdot 366 \times \frac{4 \cdot 5 \times 10^{14}}{1760 \times 36 \times 2 \cdot 54} \times \log_{10} 2$$

$$= \cdot 366 \times \frac{4 \cdot 5 \times 10^{14}}{1760 \times 36 \times 2 \cdot 54} \times \cdot 3010$$

$= 310$ megohms. *Ans.*

Example 4. The insulation resistance of a mile of insulated cable is 600 meghoms, the conductor diameter is 2 mm and the thickness of the insulating envelop is one mm. Determine the thickness of the envelop of the same material for a cable if the diameter of the conductor is 3 mm. and insulation resistance is 1200 megohms per mile.

$$\text{Insulation Res.} = \cdot 366 \frac{\rho}{l} \log_{10} \frac{r_2}{r_1}$$

$$600 = \cdot 366 \times \frac{\rho}{1} \times \log_{10} \frac{2}{1}$$

$$\therefore \quad \rho = \frac{600}{\cdot 366} \times \frac{1}{\log_{10} 2}$$

$$\text{Again} \quad 1200 = \cdot 366 \times \frac{600}{\cdot 366} \times \frac{1}{\log_{10} 2} \times \log \frac{r'_2}{1\cdot 5}$$

where r'_2 is new radius of the outer surface of insulation.

$$\text{Log} \frac{r'_2}{1\cdot 5} = 2 \log 2 = \cdot 6020$$

$$\frac{r'_2}{1\cdot 5} = 4 \text{ or } r'_2 = 6 \text{ mm.}$$

The conductor diameter being 1·5 mm. the insulating envelop is 4·5 mm. thick.

Example 5. A single core cable having a conductor diameter of 1·5 cm and overall diameter 2 9 cm. has an insulation resistance of 500 megohms per mile. Find the resistivity of the insulating material in megohms per cm. cube. The insulation resistance is to be increased to 900 megohms per mile by an additional layer of an insulating material of resistivity $6\cdot 5 \times 10^8$ megohms per cm. cube. Find the thickness of insulation to be added.

$$500 = \cdot 366 \times \frac{\rho}{1760 \times 36 \times 2\cdot 54} \times \log_{10} \frac{1\cdot 45}{\cdot 75}$$

$$\rho = \frac{500 \times 1760 \times 36 \times 2\cdot 54}{\cdot 366} \times \frac{1}{\log_{10} 1\cdot 933}$$

$$= 7\cdot 76 \times 10^8 \text{ megohms per cm. cube.}$$

$$(b) \quad 400 = \cdot 366 \times \frac{6\cdot 5 \times 10^8}{1760 \times 36 \times 2\cdot 54} \times \log_{10} \frac{r_3}{r_2}$$

$$\text{Log } \frac{r_3}{r_2} = \frac{400 \times 1760 \times 36 \times 2{\cdot}54}{6{\cdot}5 \times 10^8 \times {\cdot}366} = {\cdot}275$$

$$\frac{r_3}{r_2} = 1{\cdot}884$$

$$r_3 = 1{\cdot}884 \times 1{\cdot}45$$

Thickness of insulation $= 1{\cdot}884 \times 1{\cdot}45 - 1{\cdot}45$

$= {\cdot}884 \times 1{\cdot}45 = 1{\cdot}28$ cm. *Ans.*

Example 6. The insulation resistance of the ordinary cables used in house wiring is 600 megohms per mile at 70°F. Determine the value at 60°F if the insulating material is such that a rise of 15°F halves its resistance.

$$\text{Log } R_t = \text{Log } R_s - K(t-s)$$

$$K = \frac{\text{Log } 2}{15} = \frac{{\cdot}3010}{15} = {\cdot}02006$$

$$\text{Log } R_{60} = \text{Log } R_{70} - {\cdot}02006\,(-10)$$

$$= \text{Log } 600 + {\cdot}2006 = 2{\cdot}7782 + {\cdot}2006$$

$$= 2{\cdot}9788$$

$R_{60} = 952{\cdot}3$ megohms. *Ans.*

CHAPTER IV

OHMS LAW, RESISTANCES IN SERIES AND PARALLEL, SHUNTS, CELLS

4-1. Ohms Law.

If V is measured in volts, I in amperes, and R in ohms, then

$$\text{Current I} = \frac{V}{R} \text{ or } \frac{\text{Voltage}}{\text{Resistance}} \quad \ldots(\text{Eq. 4-1})$$

$$\text{Voltage V} = I \times R \text{ or Current} \times \text{Resistance} \quad \ldots(\text{Eq. 4-1}a)$$

$$\text{Resistance R} = \frac{V}{I} \text{ or } \frac{\text{Voltage}}{\text{Current}} \quad \ldots(\text{Eq. 4-1}b)$$

4-2. Arrangement of resistances.

(*a*) **Series.** If several resistances are connected in series so that the same current flows through each, then the total or equivalent resistance is the sum of all the resistances.

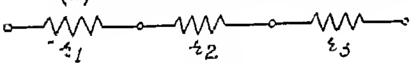
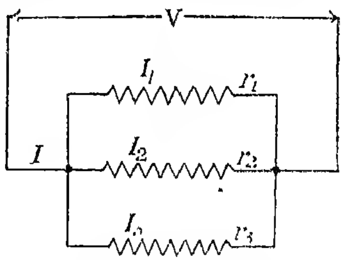


Fig. 1

Resistances in series

$$R = r_1 + r_2 + r_3 \quad \ldots(\text{Eq. 4-2})$$

(*b*) **Parallel.** When the resistances are so arranged that each forms a separate path for a part of the total current they are said to be connected in parallel.



RESISTANCES IN PARALLEL

Fig. 2

Figure shows resistances r_1, r_2, r_3 connected in parallel.

$$I_1 = \frac{V}{\quad}, \ I_2 = \frac{V}{r_2}, \ I_3 = \frac{V}{r_3}$$

$$= \frac{V}{R}$$

Resistance.

$$\frac{V}{\ _2} + \frac{V}{r_3}$$

$$\frac{1}{\ _2} + \frac{1}{r_3} \quad \ldots(\text{Eq. 4-3})$$

$\frac{1}{R}$ is called the **conductance** of the conductor and is the property of the material which conducts current.

If two conductors having resistances r_1 and r_2 are connected in parallel and I_1 and I_2 are the currents flowing through them then $\frac{I_1}{I_2} = \frac{r_2}{r_1}$

and $\quad I_1 = I \times \frac{r_2}{r_1 + r_2}$

where $\quad I = I_1 + I_2$

$$I_2 = I \times \frac{r_1}{r_1 + r_2}$$

4-3. In a cell or a dynamo

$$\text{Current } I = \frac{\;}{r+} = \frac{\text{Total pressure}}{\text{Total resistance}} \qquad \text{...(Eq. 5}a\text{)}$$

$$= \frac{V}{R} = \frac{\text{Terminal P.D.}}{\text{External Resistance}} \qquad \text{...(Eq. 5}b\text{)}$$

$$= \frac{E - V}{r} = \frac{\text{Volts drop in cell or dynamo}}{\text{Internal res. of cell or dynamo}}$$

...(Eq. 5*c*)

Example 1. A cell having an emf. of 1·5 V. and internal resistance ·5 ohm is connected in series with a cell of emf. 2V. and internal resistance one ohm. The circuit is completed by an external resistance of 5·5 ohm. Find the current flowing, p.d. across the battery and across each cell.

Total emf. $= 2 + 1{\cdot}5 = 3{\cdot}5$ V.

Total resistance $= 5{\cdot}5 + 1 + {\cdot}5 = 7$ ohms.

Current $= \frac{3{\cdot}5}{7} = {\cdot}5$ amp.

Potential drop. in the 1st. cell	$= {\cdot}5 \times {\cdot}5$	$= {\cdot}25$ V
p.d. across the 1st cell	$= 1{\cdot}5 - {\cdot}25$	$=$ **1·25 V**
Potential drop in the second cell	$= 1 \times {\cdot}5$	$= {\cdot}5$ V
p.d. across the ,, ,,	$= 2 - {\cdot}5$	$=$ **1·5 V**
p.d. across the battery	$= 1{\cdot}25 + 1.5 =$ **2·75V**. *Ans.*	

Example 2. An arc lamp is intended to operate with a current of 10 amps and terminal pressure of 48 V. What resistance should be connected in the circuit if it is fed from 110 V supply.

Volts spent in the resistance $=110-48=62$ V.

Current $=10$ amp.

Resistance required $=\frac{62}{10}=6{\cdot}2$ ohms. *Ans.*

Example 3. Three coils having resistances of 4, 10 and 20 ohms respectively are connected in parallel and the whole joined in series with a resistance of ·5 ohm to a 100 V supply. Determine the p.d. across the coils and the current in each part of the circuit.

Resistance of the 3 coils in parallel $=$ R

$$\frac{1}{R}=\frac{1}{4}+\frac{1}{10}+\frac{1}{20}=\frac{5+2+1}{20}=\frac{8}{20}$$

$R=2{\cdot}5$ ohms.

Total resistance in the circuit $=2{\cdot}5+{\cdot}5=3$ ohms.

Total current $=\frac{100}{3}=33{\cdot}3$ amps.

Drop of voltage across the coils $=2{\cdot}5\times 33{\cdot}3$

$=83{\cdot}33$ volts.

Current in Coil No. $1=\frac{83{\cdot}33}{4}=20{\cdot}82$ amps.

,, ,, ,, No. $2=\frac{83{\cdot}33}{10}=8{\cdot}33$,,

,, ,, ,, No. $3=\frac{83{\cdot}33}{20}=4{\cdot}17$,, *Ans.*

Example 4. A battery of 12 equal cells in series screwed up in a box, being suspected of having some of the cells wrongly connected is put into circuit with a galvanometer and 2 cells similar to the others. Currents in the ratio 3 and 2 are obtained according as the two extra cells are arranged to work with or against the battery. What is the condition of the battery.

Suppose n cells are connected correctly.

Then $12-n$ cells are connected wrongly.

Resistance per cell$=r.$, Emf. per cell$=$E.

$$\frac{n\mathrm{E}-(12-n+2)\ \mathrm{E}}{14r}=2$$

or $$(n-7)\times \mathrm{E}\ =14r \qquad \ldots(1)$$

$$\frac{(n+2)\ \mathrm{E}-(12-n)}{14r}=3$$

or $$(n-5)\ \mathrm{E}\ =21r \qquad \ldots(2)$$

$$\frac{n-7}{n-5}=\frac{14}{21}=\frac{2}{3}$$

$$n=11$$

So 11 cells are connected correctly and one cell connected wrongly.

Example 5. Two resistances A and B are connected in series across a 220 V supply. A voltmeter of resistance 1500 ohms is connected across A and reads 50 V, then across B and it reads 100 V. Find the resistances A and B.

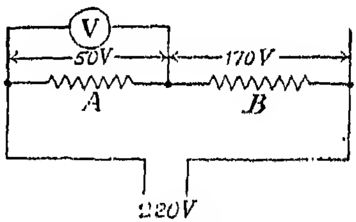


Fig. 3

Current in voltmeter$=\dfrac{50}{1500}$

$$=\frac{1}{30}\text{ amp.}$$

Current in A$=\dfrac{50}{\mathrm{A}}$

$$\therefore\quad \frac{1}{30}+\frac{50}{\mathrm{A}}=\frac{170}{\mathrm{B}}.$$

or $$\frac{1}{\mathrm{A}}=\frac{1}{50}\left(\frac{170}{\mathrm{B}}-\frac{1}{30}\right).$$

When the voltmeter is connected across B it reads 100 V and we have :

$$\frac{100}{1500}+\frac{100}{\mathrm{B}}=\frac{120}{\mathrm{A}}=\frac{120}{50}\left(\frac{170}{\mathrm{B}}-\frac{1}{30}\right)$$

or $$\frac{100}{\mathrm{B}}=\frac{120}{50}\left(\frac{170}{\mathrm{B}}-\frac{1}{30}\right)-\frac{100}{1500}$$

$$=\frac{2040}{5\mathrm{B}}-\frac{12}{150}-\frac{1}{15}=\frac{2040}{5\mathrm{B}}-\frac{11}{75}$$

or $$100=\frac{2040}{5}-\frac{11}{75}B=408-\frac{11}{75}B$$

$$B=\frac{75}{11}\times 308 \qquad =2100 \text{ ohms. } \textit{Ans.}$$

$$\frac{1}{A}=\frac{1}{50}\left(\frac{170}{B}-\frac{1}{30}\right)=\frac{1}{50}\left(\frac{170}{2100}-\frac{1}{30}\right)$$

$$=\frac{1}{50}\times\left(\frac{17-7}{210}\right) \qquad =\frac{1}{50}\times\frac{10}{210}$$

$$A=50\times 21 \qquad =1050 \text{ ohms. } \textit{Ans.}$$

Example 6. Between what resistance limits should the resistance R be operated so that the p.d. between points *a* and *b* does not exceed 10 V in the circuit shown.

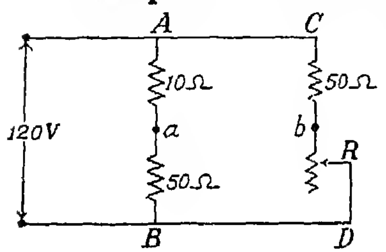


Fig. 4

Current through circuit $AB=\frac{120}{60}=2$ amps.

Voltage across 10 ohm. resistance$=10\times 2=20$V.

In order that the p.d. across points *a* and *b* may remain 10 volts the p.d. across the 50 ohm. resistance in circuit CD should be 30 V or 10 V.

When p.d. is 30 V. then current through it

$$=\frac{30}{50}=\cdot 6 \text{ amp.}$$

and $$R=\frac{120-30}{\cdot 6}=150 \text{ ohms.}$$

When p.d. is 10 V. then current through it

$$=\frac{10}{50}=\cdot 2\text{A.}$$

and $$R=\frac{120-10}{\cdot 2}=\frac{110}{\cdot 2}=550 \text{ ohms.}$$

So the resistance R can be varied between values 150 ohms. and 550 ohms.

Example 7. In the circuit shown find the value of R so that the current taken shall be 2·5A when 100 volts pressure is applied across points A and B.

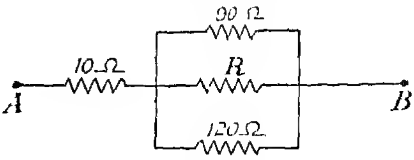


Fig. 5

Total resistance of the circuit$=\dfrac{100}{2\cdot5}=40$ ohms.

Resistance of the parallel resistances$=40-10=30$

or $$\frac{1}{30}=\frac{1}{90}+\frac{1}{R}+\frac{1}{120}$$

or $$\frac{1}{R}=\frac{1}{30}-\frac{1}{90}-\frac{1}{120}=\frac{5}{360}$$

$R=72$ ohms. *Ans.*

Example 8. In the circuit shown below the current flowing in the 8 ohm. resistance is 2·5 amp.

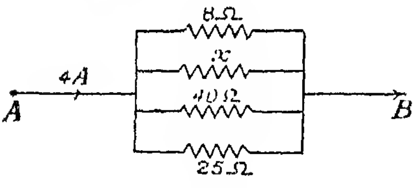


Fig. 6

Find currents in other branches and the value of the resistance x.

Voltage across points A and B

$=8\times2\cdot5=20$ V.

Current through 40 ohm. res. $=\dfrac{20}{40}=\cdot5$ amp.

" " 25 " " $=\dfrac{20}{25}=\cdot8$ "

" " x " " $=4-(2\cdot5+\cdot5+\cdot8)$

$=\cdot2$ amp.

Value of x $=\dfrac{20}{\cdot2}=100$ ohms. *Ans.*

Example 9. In the circuit shown find the voltage across the 4 ohm. resistance and the supply voltage at points A and B.

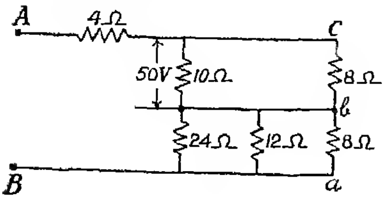


Fig. 7

Current through 8 ohm. resistance $cb = \frac{50}{8} = 6{\cdot}25$ A.

Current through 10 ohm. resistance $cb = \frac{50}{10} = 5$A.

Total Current $= 6{\cdot}25 + 5 = 11{\cdot}25$A.

Total resistance of the 3 paths connected across ab

$$\frac{1}{R} = \frac{1}{24} + \frac{1}{12} + \frac{1}{8} = \frac{6}{24}$$

R=4 ohms.

Voltage across $ab = 4 \times 11{\cdot}25$ $= 45$ volts.

,, ,, 4 ohm. resistance $= 4 \times 11{\cdot}25 = 45$ volts.

Total line voltage across AB $= 45 + 45 + 50$

$= 140$ volts. *Ans.*

Example 10. A potential divider of 160 ohms. resistance is connected across a 240 volt supply. A current of 2 amps. is required in the 20 ohm. coil. Find the position of the tapping point.

Let resistance of part AB=R.

p.d. across the coil when carrying a current of 2 amps. $= 2 \times 20 = 40$ volts.

p.d. across AB=40V.

Current in AB $= \frac{40}{R}$.

Total current in BC $= 2 + \frac{40}{R}$.

p.d. across BC $= 240 - 40$

$= 200$V

$$\frac{200}{160-R} = 2 + \frac{40}{R}.$$

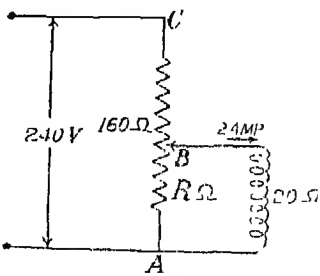


Fig. 8

$$200\,R=(160-R)\,(2R+40)$$
$$=320\,R+6400-2R^2-40R$$
$$2R^2-80R=6400$$
$$R^2-40R+400=3600$$
$$(R-20)^2=60^2$$
$$R=80 \text{ ohms.} \quad \textit{Ans.}$$

Example 11. Four resistances AB=8 ohms, BC 16Ω, CD=8 ohms, and DE=12 ohms, are connected in series across the terminals of a 150 V supply. Resistances each of 5 ohms are connected between the points B and E and between points C and E. Calculate the current flowing through the 12 ohm resistance.

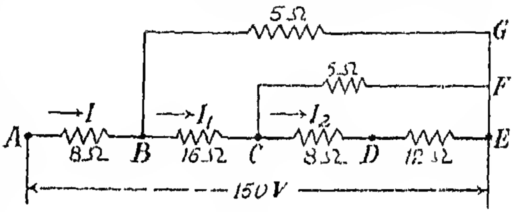


Fig. 9

Equivalent resistance of group CEF

$$=\frac{20\times 5}{20+5}=4 \text{ ohms.}$$

Equivalent resistance of group BEG

$$=\frac{20\times 5}{20+5}=4 \text{ ohms.}$$

Total equivalent resistance $=4+8=12$ ohms.

Current in $AB=I=\frac{150}{12}$ $\qquad =12{\cdot}5$ amps.

Current in $BC=I_1=12{\cdot}5\times\frac{5}{25}$ $\qquad =2{\cdot}5$ amps.

Current in $DE=I_2=2{\cdot}5\times\frac{5}{25}$ $\qquad ={\cdot}5$ amp. *Ans.*

Example 12. The leads from a dynamo are connected across 55 storage cells in series and 100 metal filament glow lamps in parallel. Each cell has an emf. of 2 volts and a resistance of ·0024 ohm. Current taken by each lamp is ·75 amp. If the

leads have a resistance of ·08 ohm. and the voltage at the load end is 120 V find :

(*a*) The charging current of the battery.

(*b*) The hot resistance of each lamp.

(*c*) The p.d. at the dynamo terminals.

Current in 100 lamp $= 75$ amp.

Emf. of 55 cells $= 110$V.

Resistance ,, ,, $= 55 \times \cdot 0024 = \cdot 132$ ohms.

Charging current of battery $= \dfrac{120-110}{\cdot 132} = 75 \cdot 8$ amp.

Hot resistance of lamp $= \dfrac{120}{\cdot 75} = 160$ ohms.

Total current $= 75 + 75 \cdot 8 = 150 \cdot 8$ amps.

Drop of Voltage in the leads $= 150 \cdot 8 \times \cdot 08$

$= 12 \cdot 1$ volts.

p.d. at the dynamo terminals $= 120 + 12 \cdot 1$

$= 132 \cdot 1$ volts. *Ans.*

Example 13. A battery of 60 secondary cells in series whose emf. has fallen to the minimum value of 1·8 volts per cell is to be charged at a constant current. If the capacity of the battery is 100 amp-hour at the 10 hour rate and its resistance is ·02 ohm. per cell, how must the applied voltage be varied during charge. Max. voltage per charged cell is 2·5 volts.

(*I*) If the supply had a constant pressure of 200 V what must be the values of the regulating resistance at the beginning and at the end of the charge. Assume charging current to be equal to the normal discharge current.

Resistance of battery $= 60 \times \cdot 02 = 1 \cdot 2$ ohms.

Current $= \dfrac{100}{10} = 10$ amps.

Emf. at beginning $= 60 \times 1 \cdot 8 = 108$ volts

,, ,, the end $= 60 \times 2 \cdot 5 = 150$,,

Volts drop in the battery $= 1 \cdot 2 \times 10 = 12$ volts.

p.d. across battery $= 108 + 12 = 120$ V at the start

and $150 + 12 = 162$ volts at the end.

Voltage drop in the beginning in the regulating resistance
$$=200-120=80\text{V}.$$

Value of regulating resistance at the beginning
$$=\frac{80}{10}=8 \text{ ohms.} \quad \textit{Ans.}$$

Voltage drop in the resistance in the end
$$=200-162=38 \text{ volts.}$$

Value of resistance at the end $=\frac{38}{10}=3{\cdot}8$ ohms. *Ans.*

Example 14. If a battery of 55 cells in series each having an emf. of 2·2 V and a resistance of ·05 ohm be giving the maximum power to an external circuit, what is the current flowing and by how much per cent will the power given to the outside circuit be reduced if the circuit be altered so that the current is reduced by 20%.

(*b*) If the external resistance consists of a simple resistance, what is the value of the resistance when it receives maximum power, and by how much percentage will the power given to the external circuit be reduced if the resistance is (*i*) 50% smaller (*ii*) 40% larger than that which corresponds with maximum power.

$\frac{nE}{nr+R}=I$	n=No. of cells
$nE-nr\ I=IR=V$	E=emf. per cell
$nEI-nr\ I^2=VI=W$	r=Internel res. per cell
When W is maximum	I=Current
$\frac{dW}{dI}=0.$	V=Voltage across R.
	W=Power in Ext. circuit.

$$nE-2\,nrI=0$$

$$E=2\,rI$$

$$I=\frac{2{\cdot}2}{2\times{\cdot}05}=22 \text{ amps.}$$

$$20\% \text{ of } 22 \text{ amps.}=\frac{22}{5}=4{\cdot}4 \text{ amps.}$$

Current $=22-4{\cdot}4=17{\cdot}6$ amps.

Resistance $= \dfrac{55\times 2{\cdot}2}{17{\cdot}6} = 6{\cdot}87$ ohms.

External resistance $= 6{\cdot}87 - 55\times{\cdot}05 = 4{\cdot}12$ ohms.

Power in outside circuit $= 17{\cdot}6^2\times 4{\cdot}12$

$= 1277{\cdot}76$ watts.

Power in the first case $= 55\times 2{\cdot}2\times 22 - 55\times{\cdot}05\times 22^2$

$= 1331$ watts.

Reduction in power $= 1331 - 1277{\cdot}6 = 53{\cdot}24$ watts.

Percentage reduction $= \dfrac{53{\cdot}24}{1331}\times 100 = 4\%$ *Ans.*

(*b*) (*i*) The value of R is 2·75 ohms, when it receives maximum power.

When R is 50% less it becomes 1·375 ohms.

Current $= \dfrac{55\times 2{\cdot}2}{2{\cdot}75+1{\cdot}375} = \dfrac{121}{4{\cdot}125} = 29{\cdot}33$ amps.

Power $= 29{\cdot}33^2\times 1{\cdot}375 = 1183$ watts.

Reduction in power $= 1331 - 1183 = 148$ watts.

$= \dfrac{148}{1331}\times 100 = 11{\cdot}12\%$ *Ans*

(*ii*) When R is 40% more it becomes

$2{\cdot}75\times 1{\cdot}4 = 3{\cdot}85$ ohms.

Current $= \dfrac{121}{2{\cdot}75+3{\cdot}85} = \dfrac{121}{6{\cdot}6} = 18{\cdot}35$ amps.

Power $= 18{\cdot}35^2\times 3{\cdot}85 = 1295$ watts.

Reduction in power $= 1331 - 1295 = 36$ watts.

$= \dfrac{36\times 100}{1331} = 2{\cdot}78\%$. *Ans.*

Example 15. 18 cells each of 1·5 V emf. and 2·2 ohm internal resistance are to supply current to an external resistance of 4 ohms. What arrangement will give the maximum current and what is the value of this maximum current.

Let the cells be arranged in m rows each with n cells in series.

For maximum current $\dfrac{nr}{m} = R$

where r=Internal resistance per cell

and R=External resistance

$$2{\cdot}2\,n = 4m$$

$$m = \frac{18}{n}$$

$$2{\cdot}2\,n = \frac{4\times 18}{\ }$$

$$\text{or } n^2 = 32{\cdot}8$$

$$\text{Take } n = 6$$

$$I = \frac{ne}{\frac{nr}{m}+R} = \frac{6\times 1{\cdot}5}{\frac{6\times 2{\cdot}2}{3}+4} = \frac{9}{8{\cdot}4}$$

$$= 1{\cdot}07 \text{ omps.} \quad \textit{Ans.}$$

Example 16. A battery having an emf. of 525 V and a resistance of ·02 ohm. is connected across the load at the end of a feeder cable, each conductor of which has a resistance of ·05 ohm. The generator bus bar voltage is 550 volts. Calculate the p.d at the terminals of the battery when the load current is (*a*) zero, (*b*) 250 amps. (*c*) 750 amps.

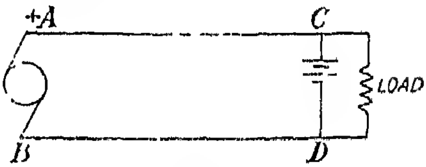


Fig. 10

(*a*) Total pressure in the circuit=550−525 V.

Current flowing in the battery= $\frac{25}{\cdot 1+\cdot 02}$ =208 amp.

Volts drop in line ·208×·1 =20·8 V

p.d. at battery terminals =550−20·8=529·2 volts. *Ans.*

(*b*) Suppose current supplied by the battery to the load is I amps.

Current supplied by the generator=250−I

p d across battery =550−(250−I)×·1=525−·02I

or 550−25+·1I =525−·02I

I=0.

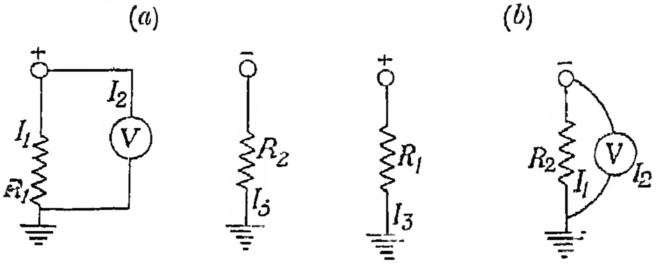
p.d. across battery =525V. *Ans.*

(*c*) Here load current=750 amp.

If the battery supplies I amps. to the load the p.d. across the battery

	$=550-(750-I)\times\cdot1$	$=525-\cdot02\ I$
or	$550-75+\cdot1\ I$	$=525-\cdot02\ I$
	$\cdot12\ I$	$=50$
	I	$=416$ amps.
	p.d. across battery	$=525-\cdot02\times416$
		$=516\cdot68$ volts. *Ans.*

Example 17. The voltage between the conductors of a two wire system is 220 volts. An insulation test on the live con-



[Fig. 11

ductors is made using a voltmeter of 25,000 ohms resistance. The voltage between positive main and earth is 40 volts and between negative main and earth is 20 volts. Find the insulation resistance of each cable to earth.

Let R_1=Insulation Resistance to earth of positive side.

R_2= „ „ „ „ „ negative side.

(*a*) $I_1+I_2 \quad =I_3$

$$\frac{40}{R_1}+\frac{40}{25000}=\frac{180}{R_2} \text{ or } \frac{2}{R_1}+\frac{2}{25000}=\frac{9}{R_2} \qquad ...(1)$$

(*b*) Test on negative cable

$$\frac{20}{R_2}+\frac{20}{25000}=\frac{200}{R_1} \text{ or } \frac{1}{R_2}+\frac{1}{25000}=\frac{10}{R_1} \qquad ...(2)$$

$$\frac{2}{R_1}=\frac{9}{R_2}-\frac{2}{25000}$$

$$\frac{10}{R_1}=\frac{45}{R_2}-\frac{10}{25000}=\frac{1}{R_2}+\frac{1}{25000}$$

or $$\frac{44}{R_2} = \frac{11}{25000}$$

$$R_2 = 100000 \text{ ohms.} \quad \textit{Ans.}$$

$$\frac{2}{R_1} = \frac{9}{100000} - \frac{2}{25000} = \frac{1}{100000}$$

$$R_1 = 200000 \text{ ohms.} \quad \textit{Ans.}$$

Example 18. A galvanometer of 25 ohms resistance is connected in series with a battery of 15 ohms. resistance. Compare the currents passing through the galvanometer before and after a shunt equal in resistance to that of the galvanometer is connected across the galvanometer.

Suppose Battery emf. $= e$.

Current in the first case $= \dfrac{e}{15+25} = \dfrac{e}{40}$ A

In the second case the resistance of the galvanometer and shunt in parallel $= \dfrac{25}{2} = 12{\cdot}5$ ohms.

Current in the circuit $= \dfrac{e}{15+12{\cdot}5} = \dfrac{e}{27{\cdot}5}$ A

Current in the galvanometer is half the above value and

$$= \frac{e}{55}$$

Currents are in the ratio $\dfrac{1}{40}, \dfrac{1}{55}$ or 11 : 8.

Example 19. The multiplying power of a shunt is 40. Determine (*a*) The joint resistance of the galvanometer and this shunt whose resistance is 20 ohms (*b*) The resistance of the galvanometer. (*c*) The current in the shunt when 2 milliamps. passes through the galvanometer.

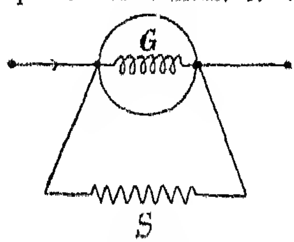


Fig. 12

Multiplying power $= \dfrac{G+S}{S}$

where G = Galvanometer resistance, S = Shunt resistance.

(a) $\frac{G+20}{20}=40$

or $G=780$ ohms. *Ans.*

(b) Combined resistance of galvanometer and shunt $=R$

$$\frac{1}{R}=\frac{1}{20}+\frac{1}{780}=\frac{40}{780}$$

$$R=\frac{780}{40}=19{\cdot}5 \text{ ohms. } Ans.$$

(c) Total current $=2\times40=80$ milliamps.

Current in the shunt $=80-2=78$ *m.* amps.

Example 20. A moving coil permanent magnet instrument needs 20 milliamps for full scale deflection and has a resistance of 3 ohms. Find the (a) shunt needed for the instrument to be used as an ammeter reading upto 10 amps. (b) Series resistance needed for it to be used as a voltmeter reading upto 250 volts.

$$I_1=I\times\frac{S}{S+G}.$$

where I_1 = Current in the instrument

I = Total current

S = Shunt resistance

G = Instrument resistance

$$\frac{20}{1000}=10\times\frac{S}{S+3} \text{ or } \frac{1}{50}=\frac{10S}{S+3}$$

$$500\,S=S+3$$

$$S=\frac{3}{499} \text{ ohms.}$$

(b) If R is the total resistance in the circuit

$$\frac{250}{R}=\frac{20}{1000}$$

or $$R=\frac{250\times1000}{20}=12500 \text{ ohms.}$$

External resistance needed $=12500-3=12497$ ohms.

Example 21. An ammeter reading correctly at 15°C has a coil of copper wire with a resistance of 2 ohms at this

temperature. The ammeter shunt has a constant resistance of ·0004 ohm. If a current of 100 amps. is flowing in the main circuit, what current flows through the ammeter coil. When the temperature is 60°C what is the percentage error in the ammeter reading. Resistance temperature coefficient for copper =·004.

Ammeter current at 15°C

$$=100\times\frac{\cdot0004}{2\cdot0004}=\frac{1}{50\cdot01}\text{ amp.}$$

$$\frac{R_{60}}{R_{15}}=\frac{R_0(1+60\alpha)}{R_0(1+15\alpha)}$$

$$R_{60}=\frac{2(1+60\times\cdot004)}{(1+15\times\cdot004)}=\frac{2\cdot48}{1\cdot06}$$

$$=2\cdot34\text{ ohms.}$$

Ammeter current at $60°C=\frac{100\times\cdot0004}{2\cdot3404}=\frac{1}{58\cdot51}$ A

$$\text{Error}=\frac{1}{50\cdot01}-\frac{1}{58\cdot51}=\frac{8\cdot5}{50\cdot01\times58\cdot51}$$

Percentage error $=\frac{8\cdot5\times100}{50\cdot01\times58\cdot51}\times50\cdot01$

$=14\cdot5\%$. *Ans.*

Example 22. A voltmeter reading upto 100 V. has a resistance of 12,000 ohms and another voltmeter reading upto 150 V. has a resistance of 15,000 ohms. If they are connected in series and joined to a 220 V. supply what does each voltmeter read.

The current flowing $=\frac{220}{12000+15000}=\frac{22}{2700}$ amp.

Voltmeter No. 1 gives a deflection 100 V when taking a current of $\frac{100}{12000}$ or $\frac{1}{120}$ amp.

Reading on Voltmeter No. $1=\frac{100}{\frac{1}{120}}\times\frac{22}{2700}$

$$=100\times120\times\frac{22}{2700}=97\frac{7}{9}\text{ volts.}$$

Reading on the other voltmeter$=220-97\frac{7}{9}$

$$=122\frac{2}{9}\text{ volts.}$$

Example 23. A 110 V. 60 watt lamp is connected in series with a 110 V. 100 watt lamp and the combination connected across 220 V. mains. Find the resistance which should be connected across the first lamp so that each lamp may get the proper current at the proper voltage.

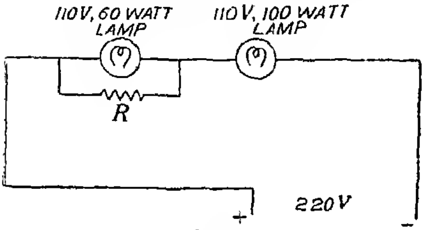


Fig. 13

Current needed for 100 watt lamp $=\frac{100}{110}=\frac{10}{11}$ amp.

" " " 60 " " $=\frac{6}{11}$ A.

Current in resistance R $=\frac{10}{11}-\frac{6}{11}=\frac{4}{11}$ A.

$$R=\frac{110}{\frac{4}{11}}=\frac{110\times 11}{4}=\frac{1210}{4}$$

$=302{\cdot}5$ ohms. *Ans.*

CHAPTER V

POWER AND ENERGY

5-1. Power and Energy

Power in watts. = Volts × Amperes

$$p = V\,I \qquad \text{...Eq. (5·1}a\text{)}$$

$$= I^2R \qquad \text{...Eq. (5·1}b\text{)}$$

$$= \frac{V^2}{R} \qquad \text{...Eq. (5·1}c\text{)}$$

And Energy taken in 't' seconds is

$$W = VIt = I^2Rt = \frac{V^2}{R}t \text{ joules or watts secs...Eq. (5-2)}$$

A larger and more convenient unit of energy than joule is the Board of Trade Unit (B.O.T. Unit).

$$\begin{aligned} 1 \text{ B.O.T. Unit} &= 1 \text{ kilowatt-hour} \\ &= 1{,}000 \text{ watts-hour} \\ &= 36{,}00{,}000 \text{ watts-secs.} \\ &= 3{\cdot}6 \times 10^6 \text{ joules} \qquad ...(i) \end{aligned}$$

The **B.O.T. Unit** is the unit of electrical energy used for commercial purposes, being the unit referred to by the public in such expressions as 'electricity at 12 nP. a unit'. Electricity supply metres record the consumption of electrical energy in kw-hr. or units.

5-2. (*a*) Relation between electrical and heat units.

Heat energy is usually measured in calories, C.H.U. or B.Th.U.

1 B. Th. U. = 5/9 C.H.U. = 252 calories.

It has been found experimentally that 1 calorie is equivalent to 4·2 joules.

$\therefore$ kw-hr. $=\dfrac{3\cdot6\times10^6}{4\cdot2}$ calories.

$=3414$ B.Th.U.

or $=1896$ C.H.U. ...(*ii*)

(*b*) **Relation between electrical and mechanical units :**

746 watts $=1$ h.p.

$\therefore$ 1 watt $=\dfrac{1}{746}$ h.p.

$\therefore$ 1 kilowatt $=\dfrac{1000}{746}=1\cdot34$ h.p. ...(*iii*)

(*c*) **Relation between mechanical and heat units :**

1 kw-hr. $=1\cdot34$ h.p. $\times3600$ secs.

$=1\cdot34\times550\times3600$ ft-lbs.

Also 1 kw-hr. $=3\cdot6\times10^6$ joules.

$\therefore$ $1\cdot34\times550\times3600$ ft. lbs. $=3\cdot6\times10^6$ joules.

$\therefore$ 1 ft-lbs. $=\dfrac{3\cdot6\times10^6}{1\cdot34\times550\times3600}$

$=1\cdot356$ joules. ...(*iv*)

Example 1. An electric iron is operated from 225 V mains, the p.d. across the iron being 220 V and the total power taken is 900 watts. Find :

(*a*) The current flowing (*b*) Resistance of the iron element (*c*) Resistance of the connecting leads (*d*) Power taken by the iron (*e*) Power spent in the connecting leads.

(*a*) Current taken from the supply $=\dfrac{900}{225}=4$ amps.

(*b*) Res. of the iron element $=\dfrac{220}{4}=55$ ohms.

(*c*) Volts drops in the leads $=225-220=5$ volts.

Resistance of leads $=\dfrac{5}{4}=1\cdot25$ ohms.

(*d*) Power taken by the iron $=220\times4$

$=880$ watts.

(*e*) Power spent in the leads $=5\times4=20$ watts.

Example 2. An electric iron taking ·5 Kw. on 220 V is found to run hotter than desired. A resistance of 10 ohms. is put in series with it to decrease the loading. (*a*) By what percentage is the power consumption reduced (*b*) what percentage of reduced power is spent in the added resistance,

Resistance of the iron $=\dfrac{220\times220}{500}=\dfrac{484}{5}$ ohms.

Resistance of the combination $=10+\dfrac{484}{5}=\dfrac{534}{5}$ ohms.

Total power consumed now $=\dfrac{220\times220\times5}{534}$

$=453$ watts.

Reduction in consumption $=500-453=47$ watts.

Percentage reduction in consumption

$=\dfrac{47\times100}{500}=9{\cdot}2\%$

Current taken now $=\dfrac{220\times5}{534}=2{\cdot}06$ amps.

Power spent in the 10 ohms resistance

$=2{\cdot}06^2\times10=42{\cdot}5$ watts.

Percentage of reduced power $=\dfrac{42{\cdot}5\times100}{453}=9{\cdot}4\%$.

Example 3. The field circuit of a motor has a resistance of 80 ohms. and carries a current of 3·5 amps. What is the p.d. across the circuit and the power being spent in it. If the circuit p.d. has a variation of $\pm6\%$, calculate the current and power spent in the two extreme cases.

p.d. $=80\times3{\cdot}5=280$ volts.

Power spent $=280\times3{\cdot}5=980$ watts.

Extreme values of p.d. $=280\pm\dfrac{280\times6}{100}$

$=296{\cdot}8,\ 263{\cdot}2$ volts.

Current in (1) $=\dfrac{296{\cdot}8}{80}=3{\cdot}71$ amps.

„ „ (2) $=\dfrac{263{\cdot}2}{80}=3{\cdot}29$ amps.

Power in (1) $=296 \cdot 8 \times 3 \cdot 71 = 1101$ watts.

" " (2) $=263 \cdot 2 \times 3 \cdot 29 = 865$ watts.

Example 4. Assuming 40 kilocalories are required per hour per cubic metre, find the cost of heating a room continuously with energy costing ·5*d.* per KWH. The room is $5 \times 4 \cdot 5$ metres in plan and 3 metres in height.

Volume of the room $=5 \times 4 \cdot 5 \times 3 = 67 \cdot 5$ cu. metres.

Heat needed per hour $=40000 \times 67 \cdot 5$ calories.

KWH used per hour $= \dfrac{40000 \times 67 \cdot 5 \times 4 \cdot 2}{3600000} = 3 \cdot 15$

Cost per hour $=3 \cdot 15 \times \cdot 5 = 1 \cdot 575d.$ *Ans.*

Example 5. A heater is required for heating a room $15' \times 15'$ and height of $12'$. The number of changes of air per hour is three. The temperature outside the room is 50°F and inside 60°F. Find the rating of the heater. Assume density of air 0·08 lb. per cu. ft. and sp. heat $= \cdot 24$.

(*b*) Find also the cost of energy consumed per month taking an 8 hour working day and 24 working days per month with energy at one anna per unit

Volume of air swept per hour $=15 \times 15 \times 12 \times 3$

$=8100$ c. ft.

Weight of air per hour $=8100 \times \cdot 08 = 648$ lbs.

Temperature rise $=60 - 50 = 10$°F

Heat needed per hour $=648 \times 10 \times \cdot 24$ B.Th.U.

$=1555 \times 252 \times 4 \cdot 2$ joules.

Power $= \dfrac{\text{joules per hour}}{3600}$

$=457$ watts. *Ans.*

(*b*) Energy consumed per month $= \dfrac{457 \times 24 \times 8}{1000}$ units.

$=87$ units

Cost per month @ -/1/- per unit = Rs. 5/7/-.

Example 6. If electrical energy for heating purposes is supplied at 2 annas per unit, find the cost of boiling a quart of water in 5 min. and the mean power required. Initial temp. of water is 15°C and efficiency of kettle is 92%.

One quart $=2{\cdot}5$ lbs. $=2{\cdot}5\times453{\cdot}6$
$=1134$ grams.

Heat reqd. $=1134\times(100-15)\times\dfrac{100}{92}$ calories

$=104800$ calories

Watts reqd. $=\dfrac{104800\times4{\cdot}2}{5\times60}=1467$ watts.

Cost of energy $=\dfrac{104800\times4{\cdot}2}{3600000}\times2={\cdot}244$ anna. *Ans.*

Example 7. What must be the power of a tin-smelting furnace in order to smelt 50 Kg. of tin per hour. Smelting temp. of tin $=235^\circ$C, Sp. heat $={\cdot}055$, Latent heat of liquification $13{\cdot}31$ calories per gram. Initial temp. of tin $=15^\circ$C and one calorie $=4{\cdot}18$ joules.

Heat required for heating $=50000\times(235-15)\times{\cdot}055$ calories

Heat for melting $=50000\times13{\cdot}31$ calories

Total heat reqd. per hour $=1270500$ calories

Heat required per sec. $=\dfrac{1270500}{3600}=353$ calories.

Power needed $=353\times4{\cdot}18=1475$ watts. *Ans.*

Example 8. If a glow lamp takes $1{\cdot}5$ watts per candle and a gas burner gives 3 c.p. per cu. ft. of gas used per hour, what must be the price per B.O.T. unit so that the cost of electric lighting may be the same as that of gas lighting with gas at 2 sh. 9 d. per 1000 cu. ft.

3 c.p. consumes one cu. ft. of gas per hour

3000 c.p. will consume 1000 c. ft. of gas per hour

Cost of 3000 candles per hour $=2$ sh. 9 d.

3000 candles consume electrical energy in one hour

$$=\frac{3000\times1{\cdot}5}{1000}=4{\cdot}5\text{ KWH.}$$

Cost of $4{\cdot}5$ KWH $=2$ sh. 9 d.

Cost per unit $=\dfrac{33\times2}{9}=\dfrac{22}{3}=7\dfrac{1}{3}$d. *Ans.*

Example 9. The load on a 110 V generator consists of 200 lamps each taking 40 watts and a 10 H.P. motor with full load efficiency of 85%. Find :

(*a*) The generator output in Kw.

(*b*) The current taken by the load.

(*c*) The H.P. of the driving engine if the efficiency of the generator is 87%.

K. Watts consumed by lamps $= \frac{200 \times 40}{1000} = 8$ Kw.

" " motor $= \frac{10 \times 746}{1000} \times \frac{100}{85} = 8{\cdot}77$ Kw.

Total generator output $= 8 + 8{\cdot}77 = 16{\cdot}77$ Kw.

Current taken by the load $= \frac{16{\cdot}77 \times 1000}{110} = 152{\cdot}5$ A

Engine H.P. $= 16{\cdot}77 \times \frac{1000}{746} \times \frac{100}{87}$

$= 25{\cdot}8$ H.P. *Ans.*

Example 10. A hoist raises a weight of 15 cwt. to a height of 400 ft. in 2 min. Find the H.P. of the motor required to drive the hoist assuming the hoist efficiency to be 75% and calculate the work done.

(*b*) The motor is connected to 220 V supply and has an efficiency of 87%. Calculate the current taken from the line.

Work done $= 15 \times 112 \times 400 = 672000$ ft. lbs.

Motor H.P. $= \frac{672000}{33000 \times 2} \times \frac{100}{75} = 13{\cdot}6$ H.P.

Watts input to motor $= 13{\cdot}6 \times 746 \times \frac{100}{87}$

Current taken from the line

$= \frac{13{\cdot}6 \times 746}{220} \times \frac{100}{87} = 52$ amps.

Ans.

Example 11. A motor drives a pump and takes 20 Kw. from the supply. The efficiency of the motor is 88% and that of the pump 85%. Find how many gallons of water are raised per min. to a height of 100 feet and the cost of running the set for 24 hours if energy for power purposes is supplied at one anna per unit.

Motor Input $=20$ Kw.

Useful work done in raising the water

$$=20\times \cdot 88\times \cdot 85=14\cdot 95 \text{ Kw.}$$

Foot lbs. of work done per min.

$$=\frac{14\cdot 95\times 1000\times 60}{1\cdot 36}=660000$$

[1·36 Joule=one ft. lb.]

Weight of water raised per min. 6600 lbs.

$=660$ gallons.

Energy consumed in 24 hours

$$=20\times 24=480 \text{ units.}$$

Cost per day $\dfrac{480}{16}=$ Rs. 30.

Example 12. A generator feeds a 100 H. P. 500V D. C. motor situated 800 yds. away. The motor efficiency at full load is 91% under which conditions the generator is found to be supplying 90 KW. Find the voltage of the generator and the weight of copper in the line.

A cubic in. of copper weighs ·318 lb.

Specific resistance of copper $=\cdot 66\times 10^{-6}$ ohm per in. cube.

Motor Input $=\dfrac{100\times \cdot 746\times 100}{1000\times 91}=82$ Kw.

Current $=\dfrac{82000}{500}=164$ amps.

$$\frac{\text{Generator Volts}\times 164}{1000}=90$$

Generator Voltage $=\dfrac{90\times 1000}{164}=548\cdot 9$ volts. *Ans.*

Drop of voltage in the line $=548\cdot 9-500=48\cdot 9$ volts.

Resistance of the line $=\dfrac{48\cdot 9}{164}=\cdot 298$ ohm.

If a is the cross-sectional area of the conductor we have ;

$$\cdot 298=\cdot 66\times 10^{-6}\times \frac{800\times 36\times 2}{a}$$

$$\text{Weight} = a \times l \times \cdot 318 \text{ lbs.}$$

$$= \frac{\cdot 66 \times 10^{-6} \times 800 \times 36 \times 2}{\cdot 298} \times 1600 \times 36 \times \cdot 318$$

$$= 2340 \text{ lbs.} \quad \textit{Ans.}$$

Example 13. An electric battery vehicle weighs 4 tons fully loaded and has a tractive resistance on the level of 65 lbs. per ton. Calculate the output of the battery when the vehicle is running. (*a*) At 16 miles per hour on the level. (*b*) Up a 1 in 30 incline at 8 miles per hour. Take the motor efficiency as 80% in each case. (*c*) Find also the battery capacity in watt hours to enable a run of 25 miles to be made on level roads with a single charge.

(*a*) Total tractive resistance $= 65 \times 4 = 260$ lbs.

H.P. output of motor $= \dfrac{16 \times 1760 \times 3}{60 \times 60} \times \dfrac{260}{550}$

$$= \frac{32 \times 26}{75}$$

Battery output Kw. $= \dfrac{32 \times 26}{75} \times \dfrac{100}{80} \times \dfrac{746}{1000}$

$$= 10 \cdot 34 \text{ Kw.}$$

(*b*) Distance run per sec. at 8 miles an hour

$$= \frac{8 \times 1760 \times 3}{60 \times 60} = \frac{176}{15} \text{ ft.}$$

Work done per sec. in raising the car up the incline

$$= \frac{176}{15} \times \frac{1}{30} \times 4 \times 2240 \text{ ft. lbs.}$$

Kw. output of battery needed for raising the car

$$= \frac{176}{15} \times \frac{1}{30} \times 4 \times 2240 \times \frac{1}{550} \times \frac{100}{80} \times \cdot 746$$

$$= 5 \cdot 95 \text{ Kw.}$$

Kw. output of battery to run the car on the level at 8 miles per hour $= \dfrac{10 \cdot 34}{2} = 5 \cdot 17$

Total Kw. reqd. $=5\cdot17+5\cdot95=11\cdot12$ Kw.

(c) Time for 25 miles run on the level $=\frac{25}{16}$ hours.

Battery capacity required$=10\cdot34\times\frac{25}{16}\times1000$

$=16130$ watt hours. *Ans.*

Example 14. A train of 60 coal tubs each weighing 16 cwt. is hauled up an incline of 1 in 22 at a speed of 3 miles an hour by an electric locomotive. If the tractive resistance is 30 lbs. per ton and the overall efficiency of the motor and gearing is 60% find the current taken from the 500 V trolley line.

Weight of coal tubs $=\frac{60\times16}{20}=48$ tons.

Resistance to traction $=48\times30=1440$ lbs.

3 miles per hour $=264$ ft. per min.

Pull needed for raising the car up the slope

$=\frac{48\times2240}{22}=4887\cdot3$ lbs.

Total pull needed $=1440+4887\cdot3$

$=6327\cdot3$ lbs.

H.P. input to motor $=\frac{6327\cdot3\times264}{33000}\times\frac{100}{60}$

$=84\cdot5$ H.P.

Current $=\frac{84\cdot5\times746}{500}=126$ amps. *Ans.*

Example 15. An electric lift raises a load of 3 tons to a height of 150 ft. The cage weighs half a ton and the balance weight 2 tons and the time taken for either an up or downward journey is one minute. Calculate the current taken by the 440 V motor running the cage and the daily cost of energy at one anna per unit if the lift makes 100 double journeys a day. The efficiency of the installation under these conditions is 60%.

(a) When the cage moves up the weight of the cage and the load is $3\frac{1}{2}$ tons, the balance weight of 2 tons moves downwards.

So the weight moving up $= 3\frac{1}{2} - 2 = 1\frac{1}{2}$ ton.

(b) When the empty cage of $\frac{1}{2}$ ton moves downward tl balance weight of 2 tons moves upwards.

So the weight moving up $= 2 - \frac{1}{2} = 1\frac{1}{2}$ tons.

Work done per min. $= 1{\cdot}5 \times 2240 \times 150$ ft. lbs.

Input per min. $= 1{\cdot}5 \times 2240 \times 150 \times \frac{100}{60}$ ft. lbs.

Current $= \frac{1{\cdot}5 \times 2240 \times 150}{33000} \times \frac{100}{60} \times \frac{746}{440}$

$= 43$ Amp.

B.O.T. units consumed in 100 double journeys

$= \frac{43 \times 440}{1000} \times \frac{200}{60} = 63$ units

Cost $= \frac{63}{16} =$ Rs. 3/15/- *Ans.*

CHAPTER VI

DIRECT—CURRENT CIRCUITS
KIRCHOFF'S LAWS—BASIC THEOREMS

6-1. Electric circuits that are more complex than those considered in the previous chapter, and particularly those which contain more than one source of emf., are solved readily by the application of Laws and theorems that relate to the several currents, emf.'s and resistance voltages in such circuits. Some such important laws and theorems have been explained below.

1. Kirchoff's Laws. The two laws are :

(*a*) In any network of conductors, carrying currents, the algebraic sum of currents at any point is zero.

(*b*) The algebraic sum of all changes of pressure round any closed mesh is zero.

Application

(*a*) Assume the values of currents and their directions in some branches of the network as I_1, I_2 etc. The currents in other branches can then be found out by the first law. For example, consider the circuit of Ex. 1. Let currents in AB, and CD be I_1, and I_2 respectively in the directions shown by arrow heads in the diagram. Then considering point *x*, the current in EF, according to the first law, would be I_1+I_2 in direction F to E.

It may be pointed out here that the choice of the current directions is only arbitrary and may or may not be correct ; an incorrectly assumed current direction will merely yield a negative answer for the quantity although the numerical result will be correct.

(*b*) After having assumed currents, go around each mesh to find the drops of voltage or rises of voltage due to resistance drops or e.m.f's in the circuit. If we move in the direction of the flow of the current there is a fall of voltage and if we move in the direction against the current flow there is rise of pressure. On equating rise of voltage to fall of voltage we will get an

equation for each mesh and the solution of such equations will give the values and directions of the currents, I_1, I_2 etc.

Care should be taken to go round all the meshes in the same directions clockwise or anticlockwise.

2. Thevenin's theorem :

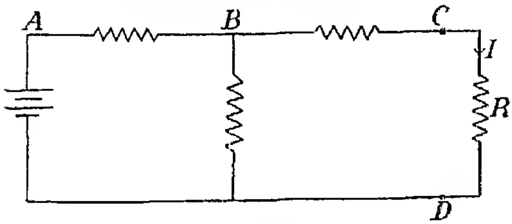


Fig. 14(a)

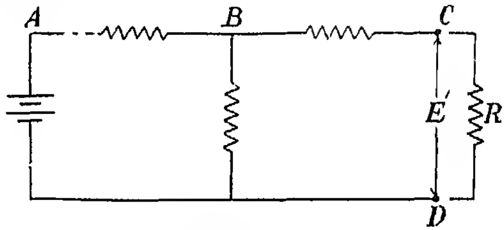


Fig. 14(b)

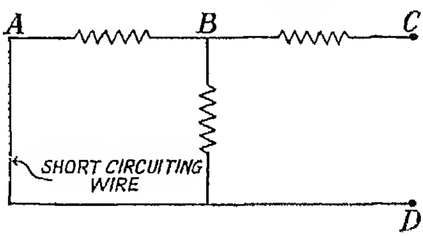


Fig. 14(c)

Statement

If a resistance R ohms is connected across points C and D in a network as shown in Fig. 14(*a*) the current flowing through R will be

$$I = \frac{E'}{R+R'}$$

where E'=p.d. across points C and D with R opened out [Fig. 14(*b*)].

R′ =Resistance of the circuit measured across C and D with R opened out, and source of e.m.f. replaced by its internal resistance (or a wire of negligible resistance if internal resistance is zero) [*see* Fig. 14(*c*)].

3. Superposition Theorem :

In a network energized by two or more sources of e.m.f., the current in any resistor, is equal to the algebraic sum of the separate currents in that resistor, assuming that each source of e.m.f., acting independently of the others, is applied separately in turn while the others are replaced by their respective internal resistances (which, if zero, would mean short-circuiting the two points.)

6-2. Delta system of resistances replaced by star system and ***vice versa.***

Sometimes resistors are connected to form very complex networks ; so that the common rules applicable to simple series and parallel circuits cannot be used for the calculation of equivalent resistances, branch currents and voltage drops. Under such conditions it is generally necessary to transform all or parts of complex circuits into electrically equivalent circuits much simpler.

Two elemental arrangements of resistors within and parts of larger networks that are frequently responsible for the diffi-

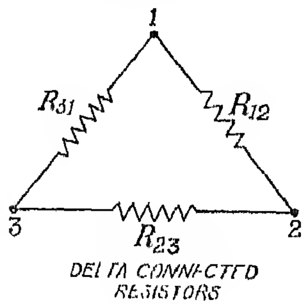


Fig. 15(*a*)

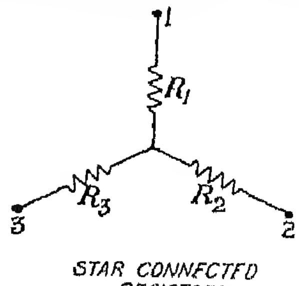


Fig. 15(*b*)

culties indicated, are Delta (△) connected resistors and Star (λ) connected resistors ; star and delta connections of resistors are shown above. The transformation of a delta into an equivalent

star or of a star into an equivalent delta may render a circuit simpler to handle. Expressions useful for such conversions are given below :

Equivalent star resistances for Fig. 15(a) are :

$$R_1 = \frac{R_{12} \times R_{31}}{\Sigma R_n}$$

$$R_2 = \frac{R_{23} \times R_{12}}{\Sigma R_n}$$

$$R_3 = \frac{R_{31} \times R_{23}}{\Sigma R_n}$$

Where $\Sigma R_n = R_{12} + R_{23} + R_{31}$

Equivalent Delta system for star system of Fig. 15(b) is :

$$R_{12} = \frac{\Sigma R_0}{R_3}$$

$$R_{23} = \frac{\Sigma R_0}{R_1}$$

$$R_{31} = \frac{\Sigma R_0}{R_2}$$

Where $\Sigma R_0 = R_1 R_2 + R_2 R_3 + R_3 R_1$

Example 1. A battery of 10 V and internal resistance ·5 ohm is connected in parallel with one of 12 V and internal

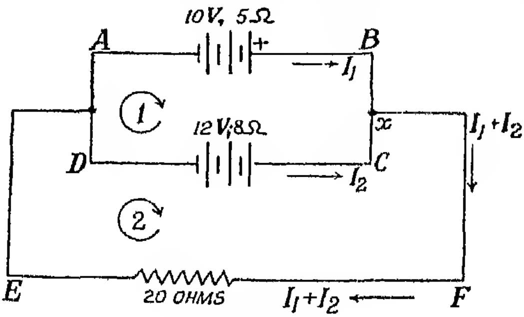


Fig. 16

resistance ·8 ohm. The terminals are connected by an external resistance of 20 ohms. Find the current in each branch.

Assume the currents I_1 and I_2 as shown.

The current in the path $FE = I_1 + I_2$.

Take Mesh No. 1. If we travel clockwise round this mesh starting from point A we have

Drop of voltage in AB due to IR $= \cdot 5\ I_1$

Rise of voltage in CD ,, ,, ,, $= \cdot 8\ I_2$

The 10 V emf. is a rise and the 12 V emf. is a drop.

Equating rise of voltage and fall of voltage

$$\cdot 5\ I_1 + 12 = \cdot 8\ I_2 + 10$$

or $$\cdot 5\ I_1 - \cdot 8\ I_2 = -2 \qquad \ldots(1)$$

For mesh No. 2 DCFE, starting from point D

$$\cdot 8\ I_2 + 20(I_1 + I_2) = 12$$

or $$20\ I_1 + 20 \cdot 8\ I_2 = 12 \qquad \ldots(2)$$

(1)×40 $$20\ I_1 - 32\ I_2 = -80 \qquad \ldots(3)$$

(2)−(3) $$52 \cdot 8\ I_2 = 92$$

$$I_2 = 1 \cdot 742 \text{ amps.}$$

Substitute the value of I_2 in (1)

$$\cdot 5\ I_1 - \cdot 8 \times 1 \cdot 742 = -2$$

$$I_1 = -1 \cdot 2128 \text{ amps.}$$

$$I_1 + I_2 = \cdot 5292 \text{ amp.}$$

$-ve$ value of I_1 indicates that the actual direction of flow of I_1 is opposite to that assumed. We can also say that battery 1 is discharging and battery 2 is being charged.

Example 2. 12 primary cells each having an emf. of 1·5V and internal resistance ·25 ohm are connected 6 in series and 2 such rows in parallel. The external circuit is closed through a resistance of ·5 ohm.

(*a*) Find the total current and the current per branch.

(*b*) If one cell from one of the branches is removed what is the value of these currents.

(*c*) Find the value of the circulating current when the external resistance is disconnected with the eleven cells arranged as in (*b*).

(*a*) Resistance of each path $= 6 \times \cdot 25 = 1 \cdot 5\ \Omega$

Emf. of each path $= 9$V

Resistance of the two paths in parallel

$$= \frac{1 \cdot 5}{2} = \cdot 75\ \Omega$$

Current in the external resistance$=\dfrac{9}{\cdot75+\cdot5}=7\cdot2$ A

Current per path $=\dfrac{7\cdot2}{2}=3\cdot6$ amps. *Ans.*

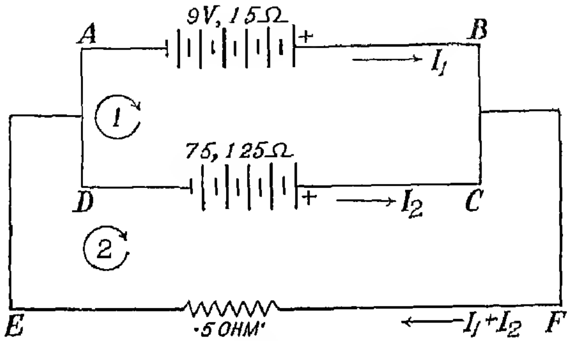


Fig. 17

(*b*) Assume currents I_1, I_2 flowing as shown in the figure.

Current in the ·5 ohm. resistance $=I_1+I_2$

Mesh (1)

$$1\cdot5\ I_1+7\cdot5=1\cdot25\ I_2+9$$

or $$1\cdot25\ I_2-1\cdot5\ I_1=-1\cdot5 \quad \ldots(1)$$

Mesh (2)

$$1\cdot25\ I_2+\cdot5(I_1+I_2)=7\cdot5$$

or $$1\cdot75\ I_2+\cdot5\ I_1=7\cdot5 \quad \ldots(2)$$

(2)×3 $$5\cdot25\ I_2+1\cdot5\ I_1=22\cdot5 \quad \ldots(3)$$

Add (1) and (3) $$6\cdot5\ I_2=21$$

$$I_2=3\ 23 \text{ amps.}$$

Substitute in (1)

$$1\cdot25\times3\cdot23-1\cdot5\ I_1=-1\cdot5$$

$$1\cdot5\ I_1=5\cdot54$$

$$I_1=3\cdot69 \text{ amps.}$$

$$I_1+I_2=3\ 69+3\cdot23=6\cdot92 \text{ amps.} \quad \textit{Ans.}$$

(*c*) Resultant emf. in the circuit $=9-7\cdot5=1\cdot5$V

Total resistance in ,, ,, $=1\cdot5+1\cdot25=2\cdot75\Omega$

Circulating current $=\dfrac{1\cdot5}{2\cdot75}=\cdot54$ amp. *Ans.*

Example 3. A battery of 50 cells in series each having an emf. of 2 V and internal resistance 001 ohm is connected in parallel with a battery of 45 similar cells in series with a resistance of ·18 ohm. The terminals are connected by an external resistance of 1·2 ohm Find the current in each branch.

Emf. of 1st battery $=50\times 2=100$ V

Internal res. of 1st battery $=\cdot 05$ ohm.

Emf. of 2nd battery $=45\times 2=90$ V

Internal res of 2nd battery $=\cdot 045$ ohm.

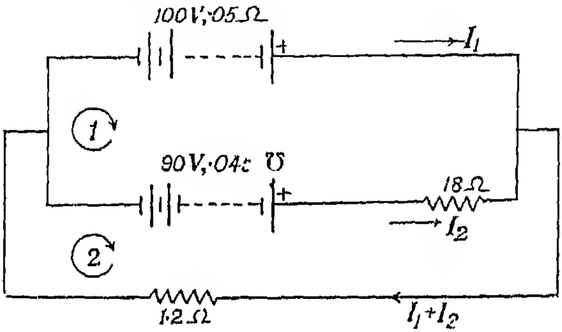


Fig. 18

Mesh (1)

$$\cdot 05\, I_1+90=100+\cdot 225\, I_2$$

$$\cdot 05\, I_1-\cdot 225\, I_2=10 \qquad \ldots(1)$$

or

Mesh (2)

$$\cdot 225\, I_2+1\cdot 2(I_1+I_2)=90$$

or $$1\cdot 2\, I_1+1\cdot 425\, I_2=90 \qquad \ldots(2)$$

(1) × 24 $$1\cdot 2\, I_1-5\cdot 4\, I_2=240 \qquad \ldots(3)$$

(2) − (3) $$6\cdot 825\, I_2=-150$$

$$I_2=-22 \text{ amp.}$$

or 22 amp. is charging current through the 90 V battery.

Substituting in (1)

$$\cdot 05\, I_1-\cdot 225\times-(22)=10$$

$$\cdot 05\, I_1=5\cdot 05$$

$$I_1=101 \text{ amp.},$$

is the discharge from 10 V. battery.

Example 4. A battery having an emf. of 110V and internal resistance ·2Ω is connected in parallel with another battery of

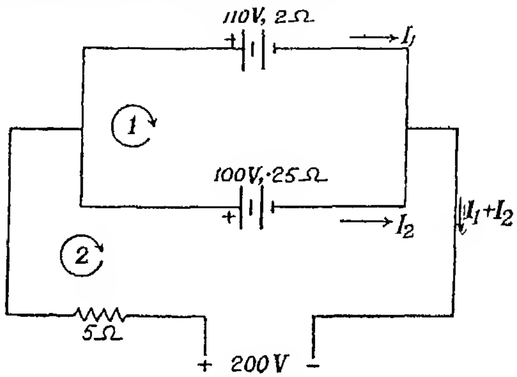


Fig. 19

emf. 100V and internal resistance ·25Ω. The two batteries in parallel are placed in series with a regulating resistance of 5 ohms. and connected to 200 V mains. Find the value and direction of current in each battery and the total current taken from the supply.

$$\cdot 2\, I_1 - \cdot 25\, I_2 + 110 - 100 = 0$$

or $$\cdot 2\, I_1 - \cdot 25\, I_2 + 10 = 0 \qquad \text{.. (1)}$$

$$\cdot 25\, I_2 + 5(I_1 + I_2) + 100 = 200$$

or $$5\, I_1 + 5\,25\, I_2 - 100 = 0 \qquad \text{.. (2)}$$

(1)×25 $$5\, I_1 - 6\,25\, I_2 + 250 = 0 \qquad \text{. (3)}$$

(3)−(2) $$-11\cdot 5\, I_2 + 350 = 0$$

$$I_2 = \frac{350}{11\,5} = 30\cdot 4 \text{ amp.}$$

Substituting in (1)

$$\cdot 2\, I_1 - \cdot 25 \times 30\cdot 4 + 10 = 0$$

$$I_1 = -12 \text{ A}$$

$$I_1 + I_2 = 30\cdot 4 - 12 = 18\cdot 4 \text{ A.}$$

12 amps. discharge from 110V battery, 30·4 amps. charge through 100 V battery and 18·4 amps. taken from the line.

Example 5. Five resistances AB, BC, CD, DA, and BD of 2, 4, 5, 6 and 8 ohms. are connected as shown and the points

A and C are connected to a cell of emf. one volt. The total res. of the path AEC is 10 ohms. Find the currents in the various branches of the network.

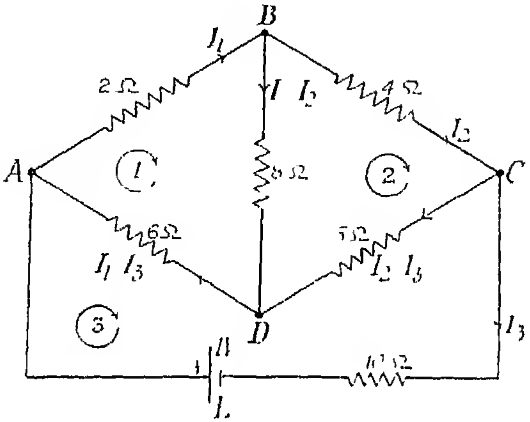


Fig. 20

Assume currents I_1, I_2, I_3 in branches AB, BC and CEA as shown. The currents in other branches are obtained from Kirchoff's first law.

There are no emfs. in Mesh (1) and (2) and we have

$$2\,I_1+8(I_1-I_2)+6(I_1-I_3)=0$$

$$4\,I_2+5(I_2-I_3)=8(I_1-I_2)$$

$$6(I_1-I_3)+5(I_2-I_3)+1=10\,I_3$$

These equations can be written as :

$$8\,I_1-4\,I_2-3\,I_3=0 \quad \ldots(1)$$

$$-8\,I_1+17\,I_2-5\,I_3=0 \quad \ldots(2)$$

$$-6\,I_1-5\,I_2+21\,I_3=1 \quad \ldots(3)$$

Add (1) and (2)

$$13\,I_2-8\,I_3=0 \quad \ldots(4)$$

(1)×3 $\quad 24\,I_1-12\,I_2-9\,I_3=0$

(3)×4 $\quad -24\,I_1-20\,I_2+84\,I_3=4$

Adding

$$-32\,I_2+75\,I_3=4 \quad \ldots(5)$$

From (4)

$$I_2 = \frac{8}{13} I_3$$

Substitute in (5)

$$-32 \times \frac{8}{13} I_3 + 75 I_3 = 4$$

$$I_3 = \cdot 0725 \text{ amp.}$$

From (4)

$$13 I_2 = 8 \times \cdot 0725 = \cdot 58$$

$$I_2 = \cdot 0446 \text{ amp.}$$

From (1)

$$8 I_1 = 4 I_2 + 3 I_3$$

$$= 4 \times \cdot 0446 + 3 \times \cdot 0725$$

$$= \cdot 3959$$

$$I_1 = \cdot 0495 \text{ amp.}$$

$$I_1 - I_3 = \cdot 0495 - \cdot 0725 = -\cdot 023 \text{ amp.}$$

$$I_2 - I_3 = \cdot 0446 - \cdot 0725 = -\cdot 0279 \text{ amp.}$$

$$I_1 - I_2 = \cdot 0495 - \cdot 0446 = \cdot 0049 \text{ amp.} \quad \textit{Ans.}$$

Example 6. Find the value of R and the current flowing through it in the network shown when current is zero in the branch OA.

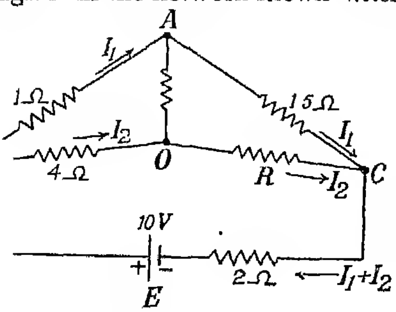


Fig. 21

As no current is flowing in OA the points O and A are at the same potential :

∴ If I_1 is the current in branch BA it is also the current in AC.

Also I_2 is current in BO and OC.

$$1 \times I_1 = 4 I_2$$

and

$$1{\cdot}5 I_1 = RI_2$$

R = 6 ohms.

In Mesh BOCE

$$10\ I_2+2\ I_1+2I_2=10$$

$$10\ I_2+8\ I_2+2\ I_2=10$$

$$I_2=\cdot 5 \text{ amp.}$$

Current through R=·5 amp. *Ans.*

Example 7. A simple potentiometer consists of a wire of resistance 20 ohms, across which a battery of 4 volts is connected. A test cell of voltage 1·5 in series with a protecting resistance of 15 ohms is tapped across the potentiometer and balance is obtained.

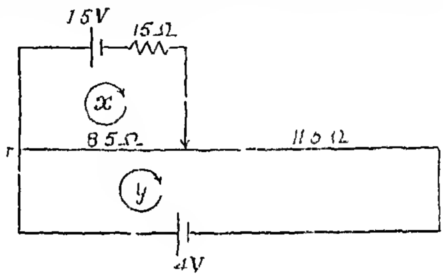


Fig. 22

Find the current flowing through the test cell when the balance is disturbed by moving the tapping key a distance along the wire towards the negative side equal to one ohm.

Resistance on the potentiometer between the test cell when balance is obtained=R.

$$\frac{1\cdot 5}{4}=\frac{R}{20} \quad \text{or} \quad R=\frac{20\times 1\cdot 5}{4}=7\cdot 5\ \Omega.$$

When tapping key is moved on the wire on the negative side equal to one ohm the res. under the test cell=8·5 ohm.

Res. outside the test cell =11·5 ohms.

$$15x+8\cdot 5(x-y)+1\cdot 5=0$$

or
$$23\cdot 5x-8\cdot 5y+1\cdot 5=0 \qquad \ldots(1)$$

$$8\cdot 5(y-x)+11\cdot 5y=4$$

$$-8\cdot 5x+20y=4 \qquad \ldots(2)$$

$$(2)\times\frac{8\cdot 5}{20} \qquad -\frac{8\ 5\times 8\cdot 5}{20}x+8\cdot 5y=1\cdot 7 \qquad \ldots(3)$$

$$(2)+(3) \qquad 19\cdot 89x+1\ 5=1\cdot 7$$

$$x = \frac{\cdot 2}{19\cdot 89}\ 10\ ma \text{ approx.} \quad \textit{Ans.}$$

Example 8. Find current in each branch in the network shown and p.d. between points O′C′ and between points B′C′ Neglect resistance of batteries.

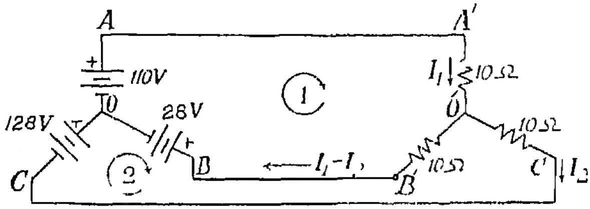


Fig. 23

Assume I_1, I_2 as shown.

Mesh (1)

$$10\ I_1 + 10(I_1 - I_2) + 28 = 110 \qquad \ldots(1)$$

Mesh (2) $\quad 10(I_1 - I_2) + 28 + 128 = 10\ I_2 \qquad \ldots(2)$

$$-10\ I_2 + 20\ I_1 = 82 \qquad \ldots(1)$$

$$20\ I_2 - 10\ I_1 = 156 \qquad \ldots(2)$$

(1) × 2 $\quad -20\ I_2 + 40\ I_1 = 164 \qquad \ldots(3)$

(2) + (3) $\quad 30\ I_1 = 320$

$$I_1 = 10\cdot 66 \text{ amps.}$$

Substitute in (2)

$$20\ I_2 - 10 \times 10\cdot 66 = 156$$

$$I_2 = 13\cdot 13 \text{ amps.}$$

p.d. between O′C′ $= 10 \times I_2 = 131\cdot 3$ V.

" " B′C′ = p.d. between BC

$= 128 + 28 = 156$ V. *Ans.*

Example 9. Find the currents in the various branches in the network shown.

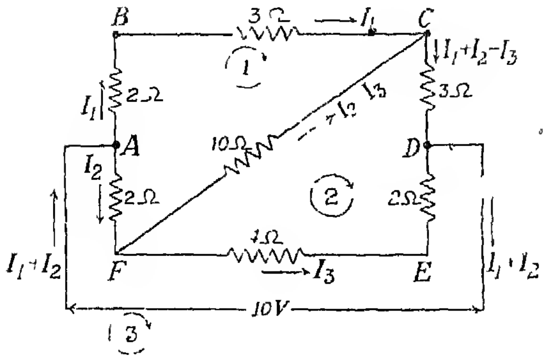


Fig. 24

$$5I_1-2\,I_2-10(I_2-I_3)=0 \qquad \ldots(1)$$

$$3(I_1+I_2-I_3)+10(I_2-I_3)-6\,I_3-0 \qquad \ldots(2)$$

$$2\,I_2+6\,I_3=1. \qquad \ldots(3)$$

Rewrite the equations :—

$$5\,I_1-12\,I_2+10\,I_3=0 \qquad \ldots(1)$$

$$3\,I_1+13\,I_2-19\,I_3=0 \qquad \ldots(2)$$

$$2\,I_3+6\,I_3=10 \qquad \ldots(3)$$

$$(2)\quad \times\frac{5}{3} \qquad 5\,I_1+21\frac{2}{3}\,I_2-31\frac{2}{3}\,I_3=0 \qquad \ldots(4)$$

$$(1)\quad -(4) \qquad -33\frac{2}{3}\,I_2+41\frac{2}{3}\,I_3=0$$

$$I_2=\frac{125}{101}I_3$$

Substitute in (3)

$$2\times\frac{125}{101}I_3+6\,I_3=10$$

$$I_3=1{\cdot}179 \text{ amps.}$$

Substitute in (3)

$$2\,I_2+6\times1{\cdot}179=10$$

$$I_2=1{\cdot}463 \text{ amps.}$$

Substitute in (1)

$$5\ I_1 - 12 \times 1{\cdot}463 + 10 \times 1{\cdot}179 = 0$$

$$5\ I_1 = 17{\cdot}556 - 11{\cdot}79 = 5{\cdot}766$$

$$I_1 = 1{\cdot}153 \text{ amps.} \quad \textit{Ans.}$$

Example 10. In the network shown find the currents in the various branches.

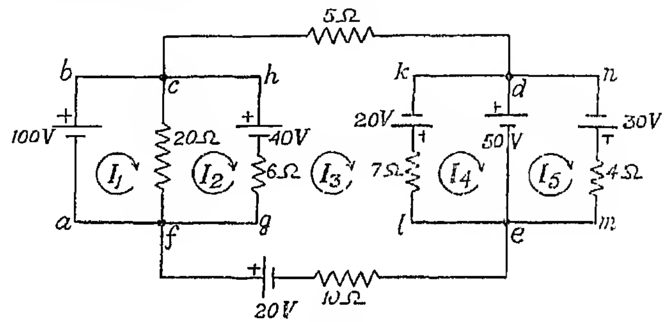


Fig. 25

Assume currents I_1, I_2, I_3, I_4, I_5 flowing in the meshes as shown.

Mesh *abcf* $\quad 20\ (I_1 - I_2) = 100$

$$I_1 - I_2 = 5 \qquad \ldots(1)$$

Mesh *fchg* $\quad 20\ (I_2 - I_1) + 6\ (I_2 - I_3) = -40$

$$-100 + 6\ (I_2 - I_3) = -40$$

$$I_2 - I_3 = 10 \qquad \ldots(2)$$

Mesh *e f g h c d k l e*

$$15\ I_3 + 6\ (I_3 - I_2) + 7\ (I_3 - I_4) = 80$$

$$15\ I_3 - 60 + 7\ I_3 - 7\ I_4 = 80$$

$$22\ I_3 - 7\ I_4 = 140 \qquad \ldots(3)$$

Mesh *kdel* $\quad 7(I_4 - I_3) = -70$

$$I_3 - I_4 = 10 \qquad \ldots(4)$$

Substitute in (3)

$$15\ I_3 + 7\ (I_3 - I_4) = 140$$

$$15\ I_3 + 70 = 140$$

$$15\ I_3 = 70$$

$$I_3 = \frac{70}{15} = \frac{14}{3} = 4\frac{2}{3} \text{ A.}$$

$$I_4 = I_3 - 10 = \frac{14}{3} - 10 = -\frac{16}{3}$$

$$= -5\frac{1}{3} \text{A}$$

Mesh *d n m e*

$$4\ I_5 = 80$$

$$I_5 = 20 \text{ amps.}$$

$$I_2 - I_3 = 10$$

$$I_2 = 10 + \frac{14}{3} = \frac{44}{3} \text{ amp.} = 14\frac{2}{3} \text{ A}$$

$$I_1 - I_2 = 5$$

$$I_1 = 5 + \frac{44}{3} = \frac{59}{3} \text{ amp.} = 19\frac{2}{3} \text{A}$$

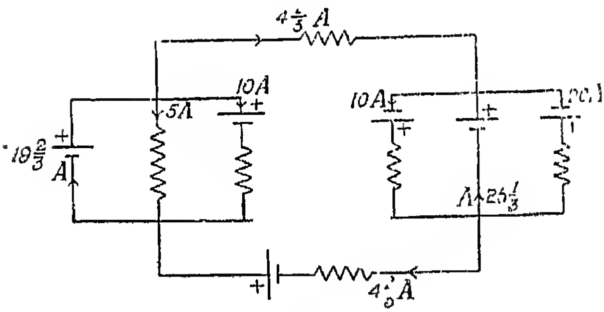


Fig. 26

Example 11.

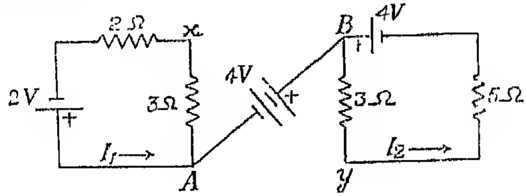


Fig. 27

In the network find p.d. across x and y.

Current $\quad I_1=\frac{2}{5}=\cdot4A$ in the direction shown

$I_2=\frac{4}{8}=\cdot5A$,, ,, ,, ,,

Point x is at a potential lower than A by $3\times\cdot4=1\cdot2$ V
Point B is at a potential higher than A by 4 V
Point y is at a potential lower than B by $3\times\cdot5=1\cdot5$ V
p.d. between points x and y

$$=1\cdot2+4-1\cdot5=3\cdot7 \text{ V.} \quad \textit{Ans.}$$

Example 12. Calculate the current in the branch XY in the circuit shown by Thevenin's theorem.

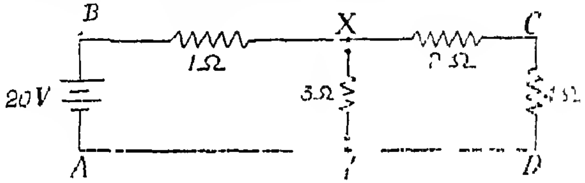


Fig. 28

With terminals XY open, the current in loop ABCD will be

$$I=\frac{20}{7} \text{ amp.}$$

Under this condition the voltage drop across 1Ω resistor would be $\frac{20}{7}\times1$ volts.

$\therefore$ the potential difference across points X and Y

$$E'=20-\frac{20}{7}=\frac{120}{7} \text{ V.}$$

To find R′, we should imagine the battery as having been removed and terminals A and B short circuited. In that case the equivalent resistance of the network across X Y (3Ω resistor still opened) would be

$$R'=\frac{1\times6}{1+6}=\frac{6}{7}\Omega.$$

∴ Current flowing through 3Ω resistor would be

$$\frac{\frac{120}{7}}{3+\frac{6}{7}}=\frac{120}{7}\times\frac{7}{27}$$

$$=\frac{40}{9} \text{ amps.} \quad \textit{Ans.}$$

Example 13. Find the current in the ammeter in the network shown by Thevenin's theorem :

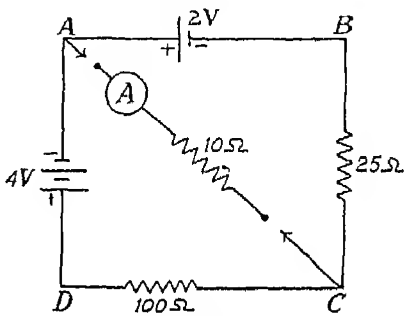


Fig. 20

With load terminals AC open, the current in loop ABCD will be

$$\frac{6}{125}\text{A.}$$

Under this condition drop of voltage along DC

$$=100\times\frac{6}{125}=\frac{24}{5}\text{V}$$

$$=4\ 8 \text{ V.}$$

If point A is assumed to be at zero potential, point C will be at a 4—4·8 *i.e.*, —·8 volts potential.

∴ Point A is at a higher potential than C by ·8 volts, so current would flow from A to C

∴ $E'=\cdot 8$ Volts.

$$R'=\frac{25\times 100}{125}=20\ \Omega$$

∴ Current through ammeter would be

$$=\frac{\cdot 8}{10+20}=\frac{\cdot 8}{30}$$

$$=\frac{8}{300} \text{ amp.} \quad \textit{Ans.}$$

Example 14. Find current in the branch FC in the network shown below by Thevenin's theorem :

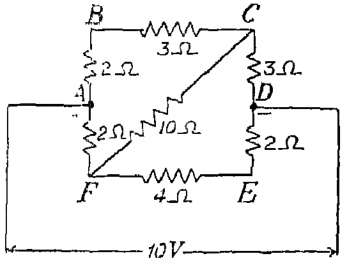


Fig. 30

With the load terminals FC open, the current in branch ABCD and similarly in AFED would be $\frac{10}{8}$ amp.$=1{\cdot}25$A.

$\therefore$ drop of voltage in ABC

$$=1{\cdot}25\times5$$
$$=6{\cdot}25\text{V}$$

And drop of voltage in

$$\text{AF}=1{\cdot}25\times2=2{\cdot}5\text{ V}$$

Evidently point F is at a higher potential by $6{\cdot}25-2{\cdot}5$ *i.e.*, $3{\cdot}75$ V

$$\therefore \qquad E'=3{\cdot}75\text{ V.}$$

To find R′ across F and C, let the source of *e.m.f.* be considered as absent and points AD short-circuited as below.

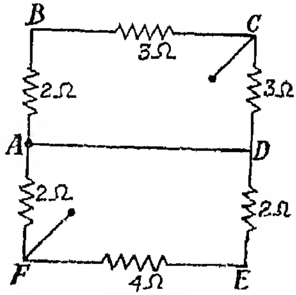


Fig. 31

Across points CA there are two resistances of 5Ω and 3Ω in parallel.

Similarly across FA, two resistances in || are 2Ω and 6Ω.

$$\therefore \quad R'=\frac{5\times3}{5+3}+\frac{6\times2}{6+2}=\frac{15}{8}+\frac{12}{8}$$
$$=\frac{27}{8}$$

Current through Resistor 10Ω would be from F to C

and
$$=\frac{3{\cdot}75}{10+\frac{27}{8}}$$
$$=\frac{3{\cdot}75\times8}{107}$$
$$=\frac{30{\cdot}00}{107}=\frac{30}{107}\text{ A.} \quad \textit{Ans.}$$

Example 15. Find current through the galvanometer when connected a cross BD by Thevenin's theorem. Galvanometer resistance is 50 ohms.

With load terminals BD open, the currents in branches ABC and ADC will be as calculated below :

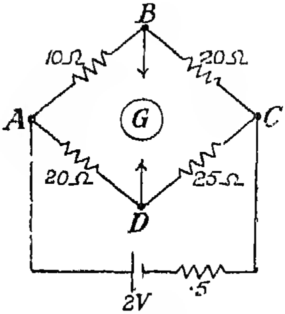


Fig. 32

Resistance of paths ABC and ADC in parallel

$$\frac{30\times45}{30+45}=18\Omega.$$

$$\text{Current in the cell}=\frac{2}{18+\cdot5}$$

$$=\frac{2}{18\cdot5}\text{A}$$

$$\therefore\quad \text{Current in ABC}=\frac{2}{18\cdot5}\times\frac{45}{75}\text{A}$$

$$\text{And Current in branch ADC}=\frac{2}{18\cdot5}\times\frac{30}{75}\text{A}$$

$$\therefore\quad \text{Drop of volts from A to B}=10\times\left(\frac{2}{18\cdot5}\times\frac{45}{75}\right)$$

$$=\frac{12}{18\cdot5}\text{V}$$

$$\text{And drop of volts from A to D}=20\times\left(\frac{2}{18\cdot5}\times\frac{30}{75}\right)$$

$$=\frac{16}{18\cdot5}\text{V}$$

∴ Point B is at higher potential and

$$E'=\frac{4}{18\cdot5}\text{V}$$

To find R′, let us consider the cell as having been removed and the terminals A and C short circuited with ·5Ω resis-

tance as shown below :

The delta ADC can be resolved into star as shown by the dotted construction :

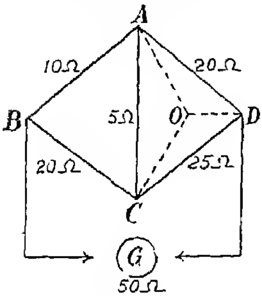


Fig. 33

$$\therefore \quad OD=\frac{25\times 20}{25+20+\cdot 5}=\frac{500}{45\cdot 5}\Omega$$

$$OC=\frac{25\times\cdot 5}{45\cdot 5}=\frac{12\cdot 5}{45\cdot 5}$$

$$OA=\frac{20\times\cdot 5}{45\cdot 5}=\frac{10}{45\cdot 5}$$

Now BA and AO are in series and

$$=10+\frac{10}{45\cdot 5}$$

$$=\frac{465}{45\cdot 5}$$

$$\text{And BCO}=20+\frac{12\cdot 5}{45\cdot 5}=\frac{922\cdot 5}{45\cdot 5}$$

$$\therefore \quad BO=\frac{\frac{465}{45\cdot 5}\times\frac{922\cdot 5}{45\cdot 5}}{\frac{465+922\cdot 5}{45\cdot 5}}$$

$$=\frac{1}{45\cdot 5}\left(\frac{465\times 922\cdot 5}{465+922\cdot 5}\right)$$

$\therefore$ BD=BO+OD in series

$$\therefore \quad R'=\frac{1}{45\cdot 5}\left(\frac{465\times 922\cdot 5}{1387\cdot 5}+500\right)=\frac{810}{45\cdot 5}=17\cdot 8\Omega.$$

$\therefore$ Current through galvanometer is

$$=\frac{\frac{4}{18\cdot 5}}{17\cdot 8+50}$$

$$=\frac{4}{18\cdot 5\times 67\cdot 8}$$

$$=\cdot 00317 \text{ amps.}$$

Example. 16. Find resistance between points A and B in the circuit shown.

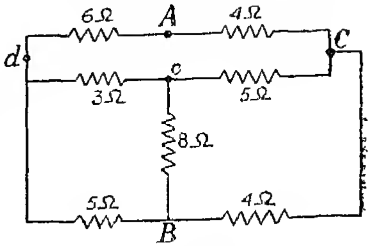


Fig. 34

The network is made of 2 sets of star connected resistances plus a resistance of 8 ohms across B and e. The star connected resistances are re-arranged below.

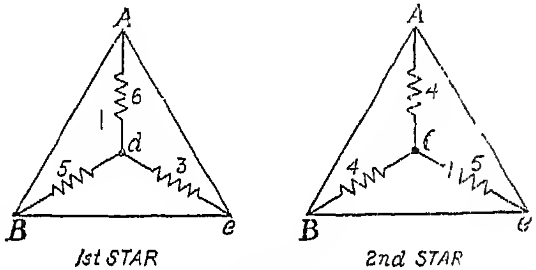


Fig. 35

To convert them into equivalent delta resistances

1st Star	2nd Star
$\Sigma R_0 = 30+15+18=63$	$\Sigma R_0=16+20+20=56$
$R_{AB} = \frac{63}{3} = 21\Omega$	$R_{AB} = \frac{56}{5} = 11{\cdot}2\Omega$
$R_{Be} = \frac{63}{6} = 10{\cdot}5\Omega$	$R_{Be} = \frac{56}{4} = 14\Omega$
$R_{eA} = \frac{63}{5} = 12{\cdot}6\Omega$	$R_{eA} = 14\Omega$

So the circuit reducces to delta form with

Two resistances in porallel between A and e

" " " " " A " B

and 3 " " " " B " e

Equivalent of R_{eA} of the 1st star and R_{eA} of the 2nd star

$$R_e A = \frac{12{\cdot}6 \times 14}{26{\cdot}6} = 6{\cdot}63 \text{ ohms.}$$

Equivalent of R_{AB} of the 1st star and R_{AB} of the 2nd star

$$R_{AB} = \frac{21 \times 11{\cdot}2}{32{\cdot}2} = 7{\cdot}3 \text{ of the 2nd star}$$

Equivalent of all the resistances between points B and e.

$$\frac{1}{R_{Be}} = \frac{1}{8} + \frac{2}{21} + \frac{1}{14} = \frac{7}{24}$$

$$R_{Be} = \frac{24}{7} = 3{\cdot}43 \text{ ohms.}$$

$$\therefore \quad \text{Total } R_{AB} = \frac{7{\cdot}3 \times (6{\cdot}63 + 3{\cdot}43)}{7{\cdot}3 + 6{\cdot}63 + 3{\cdot}43} = \frac{73{\cdot}43}{17{\cdot}36} = 4{\cdot}23\Omega.$$

Example 17. A generator and a battery whose voltages are, 129·6 and 126 volts respectively, are connected in parallel, through two resistances 2·85Ω and 2 8Ω to deliver power to a common load of 6 ohms. Using the method of superposition, calculate the current taken by the load. Figure represents the wiring diagram of the circuit.

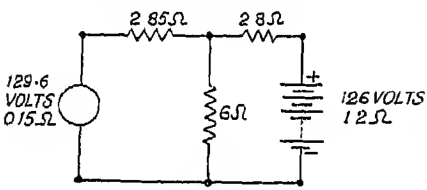


Fig. 36

Let us remove first the battery replacing it by its internal resistance. The circuit would then be supplied by the generator alone as shown in fig. 37.

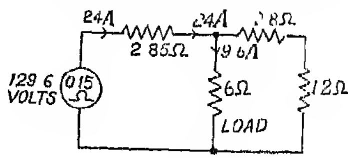


Fig. 37

Under these conditions the total resistance of the circuit would be

$$= {\cdot}15 + 2{\cdot}85 + \frac{4 \times 6}{4+6}$$

$$= 0{\cdot}15 + 2{\cdot}85 + 2{\cdot}4 = 5{\cdot}40\Omega$$

$$\therefore \quad \text{Total current} = \frac{129{\cdot}6}{5{\cdot}4} = 24 \text{ amps.}$$

$$\therefore \quad \text{current through load} = 24 \times \frac{4}{10} = 9{\cdot}6 \text{ A.}$$

Again let the dynamo be replaced by its internal resistance. The circuit is shown in fig. 38.

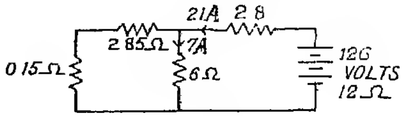


Fig. 38

The total resistance, under this condition

$$= 1{\cdot}2+2{\cdot}8+\frac{6\times 3}{6+3}$$

$=4+2=6$ ohms.

And the total current would be

$$=\frac{126}{6}=21 \text{ A}$$

This current would divide itself into two paths. The current to the load $=21\times\frac{3}{9}=7\text{A}.$

$\therefore$ Total load current when supplied both by the generator and the battery $=$ algebraic sum of individual currents

$$=+9{\cdot}6+7=16{\cdot}6 \text{ amps. } \textit{Ans.}$$

Example 18. Calculate the voltage across the 35 ohm resistor in the network shown below using theorem of superposition.

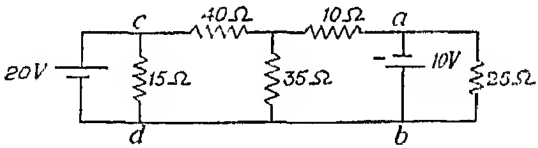


Fig. 39

Let us first find the current in the 35Ω resistance, removing the 10 volt battery and short circuiting a and b, circuit would be as in fig. 40.

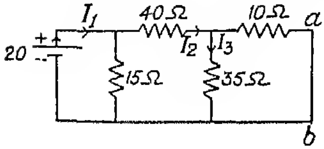


Fig. 40

10 ohms resistance is in parallel with the 35 ohms resistance.

$\therefore$ the equivalent of 10 ohms and 35Ω

$$=\frac{10\times 35}{45}=\frac{70}{9}\Omega$$

Equivalent of 40Ω and $\frac{70}{9}\Omega$ which are in series

$$=40+\frac{70}{9}=\frac{430}{9}\Omega.$$

Equivalent o $\frac{430}{9}\Omega$ resistance and 15Ω resistance, these

being in parallel $$=\frac{\frac{430}{9}\times 15}{\frac{565}{9}}$$

$$=\frac{430\times 15}{565}\text{ ohms.}$$

$$\therefore \qquad I_1=\frac{20\times 565}{430\times 15}=\frac{226}{129}\text{ amps.}$$

$$\therefore \qquad I_2=\frac{226}{129}\times\frac{15}{\frac{565}{9}}$$

$$=\frac{226\times 15\times 9}{129\times 565}$$

$$\therefore \qquad I_3=I_2\times\frac{10}{45}$$

$$=\frac{226\times 15\times 9}{129\times 565}\times\frac{10}{45}=\frac{4}{43}\text{ amp.}\downarrow$$

Now let us find current in the 35Ω resistance due to the 10V battery removing 20V battery.

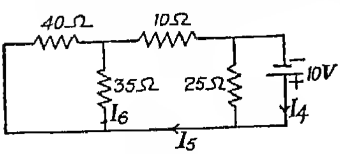


Fig. 41

Equivalent resistance of 40Ω and 35Ω which are in parallel

$$=\frac{40\times 35}{75}=\frac{56}{3}$$

Equivalent resistance of $\frac{56}{3}$ and 10Ω in series $=\frac{86}{3}\Omega$.

Equivalent resistance of $\frac{86}{3}\Omega$ and 25Ω in parallel

$$\frac{\frac{86}{3}\times 25}{\frac{86}{3}+25}=\frac{86\times 25}{161}$$

$$\therefore \qquad I_4=\frac{10\times 161}{86\times 25} \text{ ampere.}$$

$$I_5=\frac{10\times 161}{86\times 25}=\frac{25}{\left(\frac{86}{3}+25\right)}$$

$$=\frac{10\times 3}{86} \text{ ampere.}$$

$$I_6=\frac{10\times 3}{86}\times\frac{40}{75}$$

$$=\frac{8}{43} \text{ amp.} \uparrow$$

Superimposing the two currents I_3 and I_6

Current flowing thro' $35\Omega=I_6-I_3$

$$=\frac{4}{43} \text{ amp.}$$

$\therefore$ Voltage drop across 35 Ω

$$=\frac{4}{43}\times 35=\frac{140}{43} \text{ volts.}$$

Example 19. Solve Example 18 using Thevenin's theorem.

In order to find the current through the 35Ω resistor, let us remove it from the circuit. The resulting circuit diagram then reduces to as shown in fig. 42.

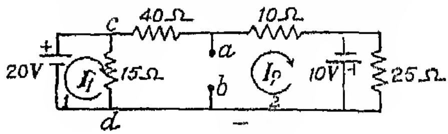


Fig. 42

Now E′ *i.e.*, the voltage across points a, b of the above circuit, can be determined as below:

Considering mesh 1,

$$15(I_1-I_2)=20 \; i.e., \; 3I_1-3I_2=4 \qquad ...(i)$$

Considering mesh 2,

$$50\ I_2+15(I_2-I_1)=10 \text{ i.e., } 13I_2-3I_1=2 \qquad \ldots(ii)$$

Solving equations (*i*) and (*ii*)

$$10\ I_2=6 \text{ or } I_2=\cdot 6 \text{ amp.}$$

$\therefore$ Voltage drop from *c* to $a=40\times\cdot 6=24$ volts.

Point *c* being at +20 volts with respect to *d*, point *a* is, therefore, at -4 volts with respect to point *d* or point *b*.

$\therefore$ $E'=4$ volts, point *b* being at a higher potential.

To find R', let us short circuit the sources of e.m.f., the circuit to diagram is shown in fig 43.

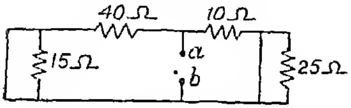


Fig 43

$$\therefore\ R'=\frac{40\times 10}{50}=8\Omega.$$

$\therefore$ Current I that will flow through the 35 Ω resistor

$$=\frac{4}{8+35}=\frac{4}{43} \text{ amp. (from } b \text{ to } a)$$

$\therefore$ Drop of voltage in this resistor

$$=\frac{4}{43}\times 35=\frac{140}{43} \text{ volts.}$$

CHAPTER VII

MAGNETIC EFFECTS OF ELECTRIC CURRENTS, MAGNETIC CIRCUITS AND ELECTROMAGNETS

7-1. Field in a conductor.

When a straight conductor carries a current, it is surrounded by concentric lines of magnetic force in the direction shown below :—

Fig. 44

The magnetic field about a wire carrying a current flowing away from the observer (cross + shows current flowing away from the observer).

The magnetic field about a wire carrying a current flowing towards the observer (dot. shows current flowing towards the observer).

The direction of the magnetic field about a wire can be easily determined by the Thumb Rule for wire which states :

"If we grasp the wire with our right hand, so that the thumb points in the direction of the current, then the fingers will curl in the direction of the lines of magnetic force." *See* fig. 45.

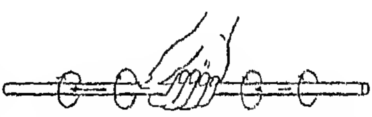


Fig. 45. The R H. Thumb Rule for finding the direction of field in a conductor.

Strength of field around a conductor carrying current is

$$H=\frac{2I}{10d} \qquad \text{... Eq. (7-1)}$$

where H=Field strength in lines per sq. cm.

I=Current in amperes.

d=Distance from conductor in centimetre.

This formula is true where wire is very long as compared to the distance d.

7-2. Field in a Solenoid.

Unless straight wires carry very large currents, the magnetic effects are weak. However, if a conductor is coiled to form a solenoid (*See* fig. 46) it is possible to magnify the magnetic effects of a current.

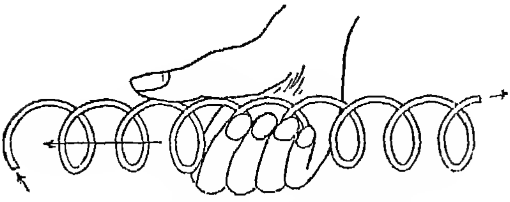


Fig. 46. R. H. Rule for finding the direction of field in a coil carrying current.

The direction of the magnetic field in a solenoid can be easily determined by the Thumb Rule for the coil which states:

"If we grasp the coil with the right hand so that the fingers point in the direction of the current in the coil, the thumb would point towards the North pole." (*See* fig. 46).

Note: The fingers should point in the direction of the current and not in the direction in which the coil was wound.

We can also find the direction with the following Rule:

Looking at the face of the solenoid if the current is flowing clockwise then that end is South pole and if the current is flowing anticlockwise that end is a North pole.

The field strength inside the solenoid is given by:

$$H=\frac{4\pi NI}{10l}$$

$$=\frac{1{\cdot}25NI}{l}$$

$$=1{\cdot}25 \text{ ampere-turns per cm.} \qquad \text{.. Eq. (7-2)}$$

where H=Field strength in lines per sq. cm.
N=Number of turns in the solenoid.
I=Current in amperes.
l=Length of the solenoid in cms.

H is called the magnetizing force.

7-3. Magnetic fields due to currents in two parallel conductors.

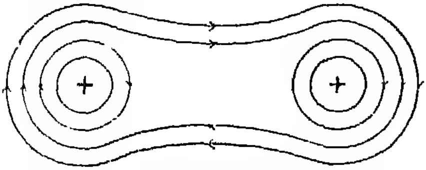


Fig. 47. Currents flowing in the same direction.

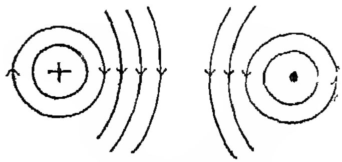


Fig. 48. Currents flowing in opposite directions.

When currents are flowing in the same direction the conductors attract one another, and when the currents are flowing in the opposite directions they repel.

7-4. Force acting on a conductor.

Any current carrying conductor when placed in a uniform magnetic field experiences a force,

$$f = \frac{HIl}{10} \text{ dynes} \qquad \ldots\text{Eq.}(7\text{-}3)$$

where H = Field strength in lines per sq. cm.

l = Length of the conductor in cms.

I = Current in Amperes.

The direction of the force acting on the conductor can be found with the Fleming's Left Hand Rule which states :

"Arrange the first and second fingers and the thumb of the left hand mutually at right angles as in Fig. 49. Point the first finger in the direction of the field, second finger in the direction of the current, then the thumb will point in the direction of the force on the conductor."

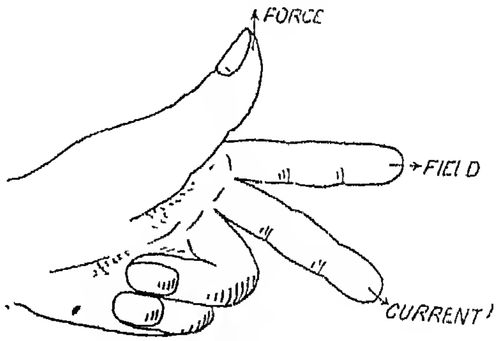


Fig. 49. Left Hand Rule.

7-5. Magnetic pressure, Mangetomotive force (MMF), Reluctance.

Just as an electric current exists due to the presence of electrical pressure in the circuit, similarly a magnetic flux, ϕ, exists due to the presence of magnetic pressure, or magnetomotive force (MMF), F, in the magnetic circuit. Some people measure F in ampere-turns but generally it is given by

$$F = 1{\cdot}25 \times \text{amp-turns.}$$

In an electrical circuit resistance opposes the flow of current whereas magnetic resistance or Reluctance, R, as it is called, opposes the flow of magnetic flux.

$$R = \frac{l}{A\mu} = \frac{\text{length}}{\text{Cross-sectional Area} \times \text{permeability}} \qquad \text{...Eq.(7-3)}$$

Permeability (μ) is the term used to express the magnetic conductance of different materials. μ for air is taken as 1. μ for iron and steel used in electrical machines has a value as high as 2,000 or even more. Permeability is a variable factor and changes widely with the flux density.

7·6. Ohm's Law for Magnetic Circuit.

$$\text{Magnetic flux} = \frac{\text{Magnetomotive Force}}{\text{Magnetic Reluctance}}.$$

or $$\text{Magnetic Lines} = \frac{1{\cdot}25 \text{ ampere-turns}}{\text{reluctance}}.$$

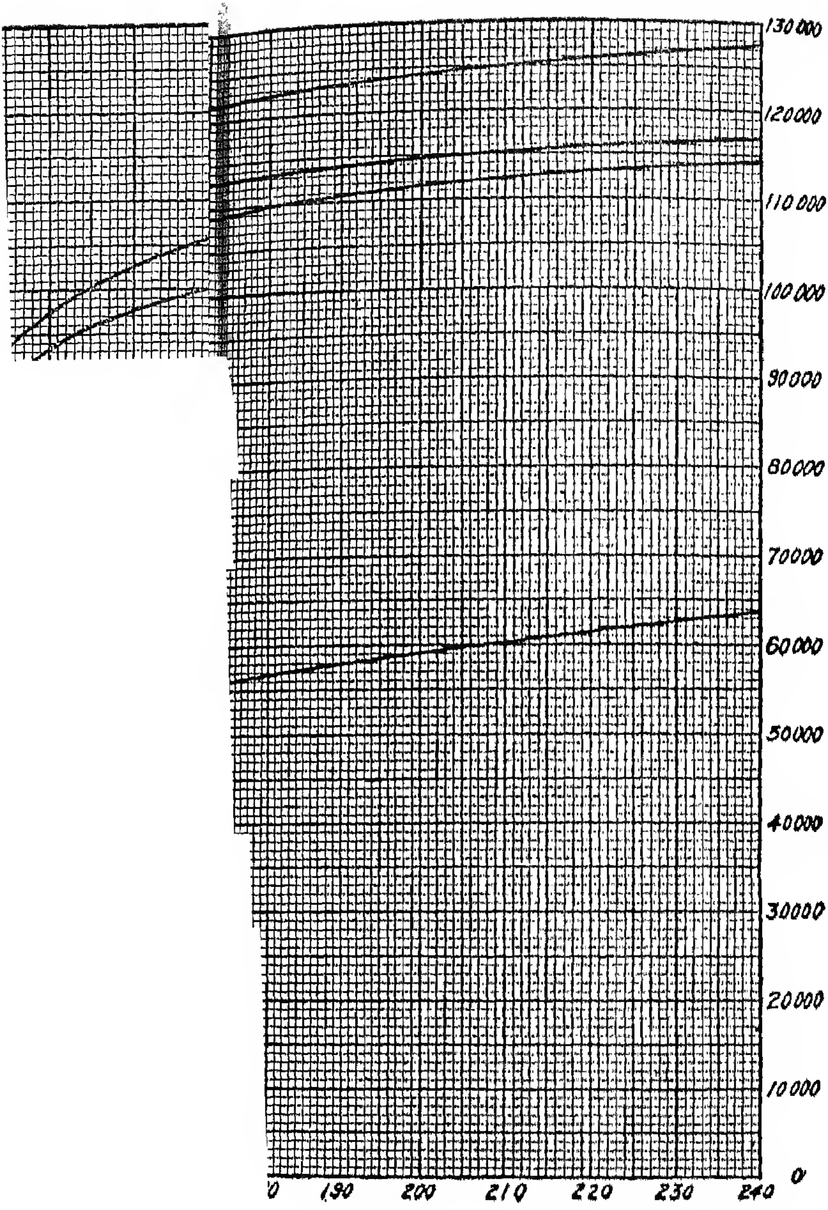
$$\phi = \frac{F}{R} \qquad \text{...Eq. (7-4)}$$

$$= \frac{1{\cdot}25NI}{\dfrac{l}{\mu A}}$$

$$= \frac{1{\cdot}25NI\mu A}{l}$$

$$\therefore \quad \frac{\phi}{A} = \frac{1{\cdot}25NI\mu}{l}$$

or B *i.e.*, flux density$$= \frac{1{\cdot}25NI\mu}{l}$$



130 000
120 000
110 000
100 000
90 000
80 000
70 000
60 000
50 000
40 000
30 000
20 000
10 000
0
190
200
210
220
230
240

Hence ampere turns needed to produce a given flux density in a material are

$$NI = \frac{Bl}{1{\cdot}25\mu}$$

$$= \frac{{\cdot}8Bl}{\mu}. \qquad \text{...Eq. (7-5)}$$

7-7. Magnetisation Curves.

The permeability and, therefore, the reluctance of a material varies very widely with the flux density. The relation between the two is generally plotted in the form of curves showing the amp. turns per inch or per cm. required to set up a given flux density in the material. These curves are known as Magnetisation curves and are very useful in the calculations of the magnetic circuit. (*See* Graph No. 50).

7-8. Hysteresis.

Hysteresis is the name given to the "lagging" of the flux density B in a piece of magnetic material behind the magnetising force H. Suppose the magnetising force acting on an unmagnetised specimen of iron is increased from zero to a certain maximum value in a series of steps and corresponding values of flux density are measured. On plotting these points a curve O*a* is obtained. If now the magnetising force is decreased to zero step by step the flux density will not decrease along the line *a*O but shall follow the line *ab*. When H has been reduced to zero, the iron is still magnetised as represented by O*b*. O*b* is a measure of the residual magnetism and to remove this, H should be increased on the negative side as represented by O*c*. Now increase the value of H on the negative side to the same maximum value as was done on the positive side, then reduce to zero and then increase it to the positive side. This gives a loop *abcdef* called the Hysteresis loop. Hysteresis results in loss of power which appears in the form of heat.

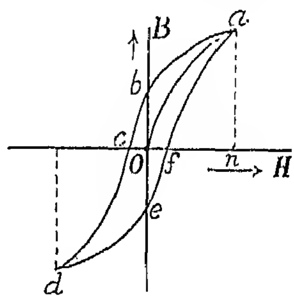


Fig. 51

It can be shown that the power wasted in this manner is proportional to the area of the loop. The shape of the hysteresis loop depends on the nature of the iron.

For this reason, the iron to be used in alternating current machines and in armatures in general, has to be selected with proper care as to its hysteresis qualities. Generally standard annealed sheet steel or annealed silicon steel is used in transformer cores, armatures, etc.

The energy lost per c.c. of iron per cycle of magnetisation is given by the formula :

$$\frac{\text{Area of the loop}}{4\pi} \text{ ergs.}$$

Dr. Steinmetz found that when the variation of the flux is sinusoidal, the hysteresis loss can be expressed conveniently by the formula

$$P=\eta\, v\, f\, B_{max}^{1\cdot 6} \times 10^{-7} \text{ watts.} \qquad \text{...Eq. (7-6)}$$

where η=hysteresis constant depending upon the material

v=volume in c.cs.

f=cycles per second.

B_{max}=Maximum flux density in lines per sq. centimeter.

7-9. Lifting Force of Electric Magnets.

$$f=\frac{B^2A}{8\pi} \text{ dynes.} \qquad \text{Eq.(7-7)}$$

where f=lifting force in dynes.

B=lines per sq. cm. in air gap.

A=area in sq. cms.

Expressed in inch units

$$f=\frac{B^2A}{72\times10^6} \text{ lbs.} \qquad \text{Eq.(7-7}a\text{)}$$

where B=lines per sq. inch. air gap.

A=area in sq. inches.

Example 1. Calculate the field density (*a*) at the surface of (*b*) 10 cms. away from a long straight isolated conductor ·5 cm. dia. carrying a direct current of 100 amps.

$$H=\frac{2I}{10d}.$$

(a) $$H=\frac{2\times 100}{10\times \cdot 25}=80 \text{ lines per sq. cm.}$$

(b) $$H=\frac{2\times 100}{10\times 10}=2 \text{ lines per sq. cm.}$$

Example 2. A wire is bent into a plane square of 10 inches side and a current of 250 amps. is circulated round it. Find the magnetic field strength at the centre of the square.

Field at P due to conductor AB

$$=\int_{-\theta_1}^{\theta_2}\frac{I}{d}\cos\theta\, d\theta$$

$$=\frac{I}{d}\left[\sin\theta_2+\sin\theta_1\right]$$

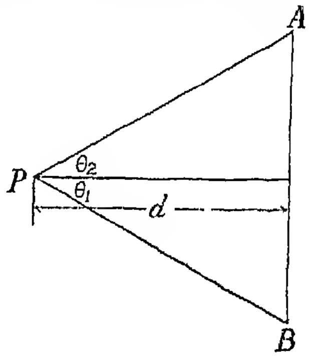


Fig. 52

Here θ_2 and θ_1 are both 45°.

Field produced at the centre by each side of the square

$$=\frac{I}{d}\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)=\frac{I}{d}\times\sqrt{2}$$

Field produced by all the 4 sides of the square

$$=4\times\sqrt{2}\times\frac{250}{10}\times\frac{1}{5\times 2\cdot 54}$$

$$=11\cdot 12 \text{ lines per sq. cm.}$$

Example 3. A conductor lying perpendicular to a magnetic field of 6,000 lines per sq. cm. is carrying a current of 20 amps. Find the force acting on the conductor in pounds per ft. run.

$$f=\frac{HlI}{10} \text{ dynes.}$$

$$=\frac{6000\times 12\times 2\cdot 54\times 20}{\times 981\times 453\cdot 6} \text{ lb.}=\cdot 825 \text{ lb.} \quad \textit{Ans.}$$

Example 4. Two parallel conductors carry currents of 150 amps. and 250 amps. respectively in the same direction. Find the force between them per foot length if their centres are 2" apart.

The field produced by the current of 150 amps. at the centre of the other conductor $=\dfrac{2\times 150}{10d}$

$$=\frac{2\times 150}{10\times 2\times 2{\cdot}54}$$

$$=\frac{15}{2{\cdot}54}\text{ lines per sq. cm.}$$

This field acts on the current of 250 amps. and produces a force of repulsion.

$$f=\frac{\mathrm{H}l\mathrm{I}}{10}\text{ dynes}=\frac{15}{2{\cdot}54}\times\frac{12\times 2{\cdot}54\times 250}{10}$$

$$=4500\text{ dynes}={\cdot}0101\text{ lb.}\quad \textit{Ans.}$$

Example 5. The coil of a moving coil ammeter has 50 turns and the average flux density in the air gap is 800 lines per sq. cm. The length of the coil side in the air gap is 2·5 cms. and the mean distance from the axis of the coil is one cm. Calculate the torque on the coil when it carries a current of 10 milliamps.

Force on each coil side $=\dfrac{\mathrm{H}l\mathrm{I}}{10}\times \mathrm{N}$

$$=\frac{800\times 2{\cdot}5}{10}\times\frac{10}{1000}\times 50=100\text{ dynes.}$$

$$\text{Torque}=100\times 1\times 2=200\text{ dyne cm.}\quad \textit{Ans.}$$

Example 6. The coil of a permanent magnet moving coil voltmeter has 60 turns of wire, an effective length of 3·5 cms., a breadth of 2 cms. and moves in a field of 900 lines per sq. cm. The range of the voltmeter is 0—300 volts and at full scale deflection the control springs exert a torque of ·5 gram. cm. Calculate the total resistance of the instrument.

$$f=900\times 3{\cdot}5\times\frac{\mathrm{I}}{10}\text{ dynes per turn.}$$

Torque $=900\times 3{\cdot}5\times\dfrac{\mathrm{I}}{10}\times\dfrac{60\times 2}{981}$

$$={\cdot}5\text{ gram cm.}$$

$$\mathrm{I}=\frac{{\cdot}5\times 10\times 981}{120\times 3{\cdot}5\times 900}=\frac{109}{8400}\text{ amp.}$$

$$\mathrm{V}=300$$

Resistance $=\dfrac{300\times 8400}{109}=23100$ ohms. *Ans.*

Example 7. An electric motor has 7000 conductors each 24 cms. long. Two-third of the total number of conductors lie under the poles and each carries a current of 10 amps. The conductors are so connected that the torque produced by all the conductors is in the same direction. The field strength is 6000 lines per sq. cm. and armature diameter is 40 cms. Find the turning moment exerted by the armature in metre kilograms.

Total length of conductors under the poles

$$=\frac{2}{3}\times 7000\times 24 \text{ cms.}=112000 \text{ cm.}$$

Force produced by all conductors

$$=6000\times 112000\times\frac{10}{10}$$

$$=67{\cdot}2\times 10^7 \text{ dynes.}$$

$$=\frac{67{\cdot}2\times 10^7}{981\times 1000}=685{\cdot}2 \text{ Kg.}$$

This force acts on a radius of $\frac{40}{2}$ cms.

Turning moment $=685{\cdot}2\times\frac{20}{100}=137$ Kg. metres. *Ans.*

Example 8. In a 4 pole direct current motor the number of conductors on the armature is 180 and there are 2·9 megalines per pole. What torque will the motor exert when a current of 50 amps. flows through each conductor ?

If d be the dia. of the armature and l its length in cms.

Average field strength $=\frac{2{\cdot}9\times 10^6\times 4}{\pi dl}$ lines per sq. cm.

Average force per conductor

$$=\frac{2{\cdot}9\times 10^6\times 4}{\pi dl}\times\frac{50}{10}\times l \text{ dynes.}$$

Torque = Force per conductor × Number of conductors × radius

$$=\frac{2{\cdot}9\times 10^6\times 4}{\pi dl}\times\frac{50}{10}\times l\times 180\times\frac{d}{2}$$

dyne cms.

$$=16{\cdot}95 \text{ Kg. metres. } \textit{Ans.}$$

Note. Actually all the conductors are not under the poles but the answer is not affected in any way by that. If the number of conductors under the poles is only taken then the field strength is proportionately increased.

Example 9. Determine the reluctance, the MMF, magnetising force and the amp.-turns necessary to produce flux densities of (*a*) 5,000 and (*b*) 10,000 lines per sq. cm. respectively in an iron core with a mean length of magnetic circuit of 60 cms. and cross-section of 20 sq. cms. Assume that the permeability has values of 2,500 and 2,000 at the above densities.

Reluctance $=\dfrac{l}{A\mu}$

(*a*) $\dfrac{60}{20\times 2500}=\cdot 0012$

(*b*) $\dfrac{60}{20\times 2000}=\cdot 0015$

MMF, $F=\text{Flux}\times\text{Reluctance}$

$=\dfrac{\phi l}{A\mu}=\dfrac{Bl}{\mu}$

(*a*) $\dfrac{5000\times 60}{2500}=120$

(*b*) $\dfrac{10000\times 60}{2000}=300$

Magnetising force H $=\dfrac{4\pi NI}{10l}=\dfrac{1\cdot 25NI}{l}=F/l$

(*a*) $\dfrac{120}{60}=2$

(*b*) $\dfrac{300}{60}=5$

Amp. turns $=\dfrac{\cdot 8\ Bl}{\mu}$

(*a*) $\cdot 8\times 120=96$

(*b*) $\cdot 8\times 300=240$

Example 10. The magnetic circuit shown is built up of iron of square cross-section 2 cm. side. Each air gap is 2 mm. wide. The permeability of part A of the circuit is 1000 and that of part B is 1,200. Each of the exciting coils has 1000 turns and the exciting current is one amp.

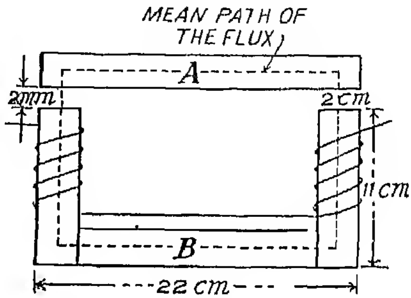


Fig. 53

Find (*a*) Reluctance of part A.

(*b*) Reluctance of part B.

(*c*) Reluctance of two air gaps.

(*d*) Total reluctance

(*e*) The MMF

(*f*) Total flux

(*g*) Flux density.

Neglect leakage and fringing.

The mean flux path is shown in the diagram.

(*a*) Reluctance of A $=\dfrac{l}{A\mu}$

$$=\frac{20}{4\times 1000}=\cdot 005 \text{ unit.}$$

[Length of path of flux in A is 20 cms.]

(*b*) Reluctance of B $=\dfrac{40}{4\times 1200}=\dfrac{1}{120}=\cdot 0083$ unit.

[Length of path of flux in B is 40 cms.]

(*c*) Reluctance of two air gaps

$$=\frac{\cdot 4}{4\times 1}=\cdot 1 \text{ unit}$$

(*d*) Total reluctance $=\cdot 1+\cdot 0083+\cdot 005=\cdot 1133$ units

(*e*) MMF$=1\cdot 25$NI $=1\cdot 25\times 1000\times 2\times 1$

$=2500$

(*f*) Total flux $\phi=\dfrac{\text{MMF}}{\text{Reluctance}}=\dfrac{2500}{\cdot 1133}=22000$ lines

(*g*) Flux density $B=\dfrac{22000}{4}$

$=5500$ lines per sq. cm. *Ans.*

Example 11. An iron ring consists of 35 cm. length of 10 sq. cm. cross-section, 25 cm. length of 20 sq. cm. cross section and 60 cm. length of 25 sq. cm. cross-section. If it is wound with 10 turns per cm. length, what current will produce a flux of 1,80,000 lines? The values of μ at densities 18000, 9000 7,200 are 163, 1,200 and 1,600 respectively.

$$NI = \cdot 8 \frac{Bl}{\mu}$$

amp. turns needed $= \cdot 8 \times \frac{18000}{163} \times 35 = 3080$

$\cdot 8 \times \frac{9000}{1200} \times 25 = 150$

$\cdot 8 \times \frac{7200}{1600} \times 60 = 216$

Total amp. turns $= 3446$

No of turns $= 120 \times 10 = 1200$

Current $= \frac{3446}{1200} = 2 \cdot 87$ amps. *Ans.*

Example 12. A cast iron ring has a cross-section of 4 sq. cm. and a length of 20 cms. Calculate the amp. turns required to drive a flux of 28,000 lines through the ring. $\mu = 100$

(*b*) Find tne additional amp. turns required if an air gap ·5 cm. wide is cut in the ring.

$$NI = \cdot 8 \times \frac{28000}{4} \times \frac{20}{100} = 1120$$

(*b*) NI for air gap $= \cdot 8 \times 7000 \times \cdot 5 = 2800$. *Ans.*

Example 13. Find the ampere turns required to produce a flux of 32,000 lines in the air gap in the magnetic circuit shown. Allow a leakage factor of 1·1 and take μ for iron to be 1,600 at this flux density. The iron is of square cross-section of 2 cm. side.

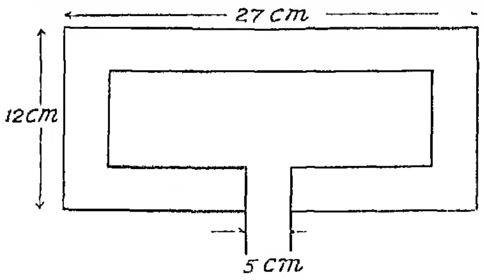


Fig. 54

Flux density in the gap $= \frac{32000}{4} = 8000$ lines per sq. cm.

Flux density in the iron $=8000\times 1{\cdot}1=8800$

Mean length of magnetic circuit of iron

$$=25+10+10+24{\cdot}5=69{\cdot}5 \text{ cms.}$$

Amp. turns for air gap $={\cdot}8\times 8000\times{\cdot}5$

$$=3200$$

Amp. turns for iron $=\dfrac{{\cdot}8\times 8800\times 69{\cdot}5}{1600}=305$

Total ampere turns $=3200+305=3{,}505$. *Ans.*

Example 14. The parallel magnetic circuit shown is a soft steel casting. A coil of 600 turns on part L carries a current of 1·5 amps. Compute the flux in each part. The thickness of the casting is 1·5″.

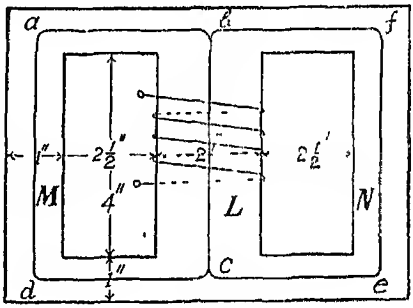


Fig. 55

The flux passes through the part L of the magnetic circuit, divides at b in 2 equal parts—one part going to N and other part to M. Then the fluxes from N and M meet at C.

The length of the path of the flux $abcd=18''$.

Amp. turns produced by the coil $=600\times 1{\cdot}5=900$

The flux density in all parts of the circuit is the same.

Amp. turns per inch $=\dfrac{900}{18}=50$

From the magnetization curve graph the flux density of soft steel casting produced by 50 amp. turns per inch is

$$=89000$$

Flux in part L $=89000\times 3=267000$ lines

Flux in part M or N $=89000\times 1{\cdot}5=133500$ lines. *Ans.*

Example 15. The figure shows the magnetic circuit of a 2 pole DC machine. The various dimensions are :—

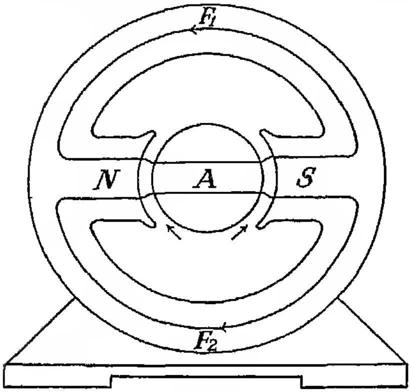


Fig. 56

Length of each cast iron pole 4·5 inches, average cross-section 16 sq. in. Nett area of armature core of annealed sheet steel is 10 sq. in. and average length of magnetic path through the armature is 3″. Length of each air gap ·125 inch and area 32 sq. in. Length of magnetic path through the cast steel frame is 18 inches and area of cross section 9 sq. in. Find the amp. turns required to send a flux of $1·6 \times 10^6$ lines through the armature.

Poles

Flux density $=\dfrac{1·6\times10^6}{16}=100{,}000$ lines per sq. in.

Amp. turns per inch (from graph) $=92$

Amp. turns needed $=92\times9$ $=828$ amp. turns

Armature

Flux density $=\dfrac{1·6\times10^6}{2\times10}=80{,}000$ lines per sq. in.

There are 2 parallel magnetic paths through the armature and only half the total flux passes through each path.

Amp. turns per inch (from graph) $=8$

Amp. turns needed $=8\times3$ $=24$

Frame

Flux density $=\dfrac{1·6\times10^6}{2\times9}=88{,}000$ lines per sq. in.

Amp. turns per in. (from graph) $=50$

Amp. turns $=50\times18$ $=900$

Gap

Flux density $=\dfrac{1{\cdot}6\times10^6}{32}=50{,}000$ lines per sq. in.

Amp. turns for both gaps

$$=\frac{{\cdot}8\times50000}{2{\cdot}54\times2{\cdot}54}\times{\cdot}25\times2{\cdot}54$$

$$=\frac{50000}{3{\cdot}2}\times{\cdot}25=3{,}890$$

Total amp. turns needed

$$=828+24+900+3890$$

$=5642$ amp. turns. *Ans.*

Example 16. Find the flux produced in a cast steel ring when 3000 ampere turns are applied to it. The ring has a cross-section of 4 sq. in. and a mean length of 25 inches. A gap ·12 in. is cut in the magnetic circuit of the ring and the average area of this gap is 5 sq. in.

The air has a much greater reluctance than iron and we assume that 80% of the amp. turns are used up to send the flux through the gap.

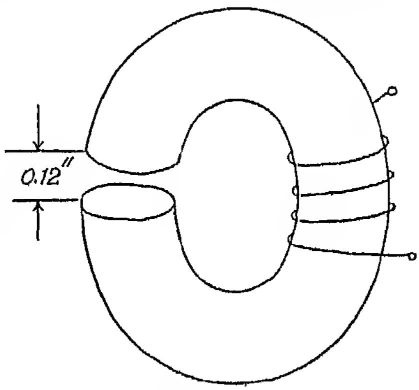


Fig. 57

Amp. turns for gap $=3000\times{\cdot}8=2400$

Amp. turns per inch $=\dfrac{2400}{{\cdot}12}=20{,}000$

Flux density in gap $=3{\cdot}2\times20000=64{,}000$ lines per sq. in.

Flux in gap $=5\times64000=320{,}000$

B in Iron $=\dfrac{320000}{4}=80{,}000$ lines per sq. in.

Amp. turns per in. (from graph)

$=35$

Amp. turns for Iron $=35\times25=875$

Total amp. turns $=3275.$

Thus we find that we have assumed too much flux density in the gap. The amp. turns needed are 275 more than are available and are $\frac{3275}{3000}\times 100=109\%$ of the available amp. turns. The flux can probably be reduced by 8% to get the correct ampere turns.

Flux density in gap $=64000\times\cdot 92=58{,}800$

Total flux $=5\times 58800=294{,}000$

Gap amp. turns $=2400\times\cdot 92=2208$

Flux density in Iron $=\frac{294000}{4}=73300$

Amp. turns per inch (from graph)

$=30$

Total amp. turns for Iron

$=30\times 25=750$

Total amp. turns $=2208+750=2958$

This gives the ampere turns about 1·4% too low. It will be sufficiently accurate to increase the flux in this ratio.

Total flux $=294000\times 101\cdot 4=298{,}000$ lines. *Ans.*

Example 17. A horse shoe magnet is formed from a wrought iron bar 18 in. long and having a cross-section of 1·5 sq. in. Exciting coils of 500 turns each are placed on each limb and connected in series. Find the exciting current that will just hold a load of 225 lbs. assuming that the two air gaps of contact are one mm. long each. Take permeability of wrought iron as 700 and the reluctance of the magnetic circuit through the load as 25% of the reluctance of the horse shoe (excluding the air gaps).

Force $f=\frac{B^2A}{8\pi}$ dynes$=\frac{B^2A}{8\pi\times 981\times 453\cdot 6}$ lbs.

$$B=3345\sqrt{\frac{f}{A}}$$

where f is in lbs., A is in sq. cms., B is in lines/sq. cm.

$$=3345\times\sqrt{\frac{225}{(3\times 6\cdot 45)}}=11{,}345$$

Amp. turns (NI) for Iron

$$=\frac{\cdot 8Bl}{\mu} \text{ where } l \text{ is in centimetres.}$$

$$=\frac{\cdot 8\times 11345\times(18\times 2\cdot 54)}{700}=593$$

NI for gap $=\cdot 8\times 11345\times\cdot 2=1820$

NI for load reluctance $=\frac{593}{4}=148$

Total NI $=2561$

Current $=\frac{2561}{1000}=2\cdot 56$ amps. *Ans.*

Example 18. A circular lifting magnet has the inner circular pole of 12″ dia. The outer annular pole has the same area as the inner one. The mean length of the magnetic path is 50″ and the permeability of steel is 1000. If it is lifting steel plates of negligible reluctance and the plates are separated ¼ from the magnet poles, find the amp. turns necessary to produce a flux density of 10,000 lines per sq. cm. Find also the weight that it can just lift with this flux density.

Mean length of steel path

$$=50\times 2\cdot 54=127 \text{ cms.}$$

NI for Iron $=\cdot 8\times\frac{10000}{1000}\times 127=1016$

Length of air gaps $=\frac{1}{4}\times 2\times 2\cdot 54=1\cdot 27$ cm.

NI for gaps $=\cdot 8\times 10{,}000\times 1\cdot 27$

$=10{,}150$

Total amp. turns needed

$=1{,}016+10{,}150$

$=11{,}166$ amp. turns

Weight that it can just lift

$$=\frac{B^2A}{8\pi} \text{ dynes.}$$

$$=\frac{10000^2\times 12^2\times 6\cdot 45\times 2}{8\pi\times 981\times 453\cdot 6}\times\frac{\pi}{4} \text{ lbs.}$$

$=5\cdot 8$ tons. *Ans.*

Example 19. Two rods of circular cross-section are placed end to end inside a solenoid. If the magnetising force of the solenoid is 10 units and the area of the contact of the rods is 4 square cm., calculate the force of attraction between the rods in lbs. The permeability of iron is 800.

Flux density in the rods

$$=10\times 800=8{,}000$$

$$f=\frac{B^2A}{8\pi}\text{ dynes}$$

$$=\frac{8000\times 8000\times 4\times 7}{8\times 22\times 981\times 453{\cdot}6}=23{\cdot}1\text{ lbs. } \textit{Ans.}$$

Example 20. In a hysteresis loop showing B plotted against H the vertical scale of B is 1″=2000 lines per sq. cm. and the horizontal scale of H is 1″=10. The area enclosed by the loop is 1·75 sq. inches. Find the hystersis loss per cycle per c.c.

One sq. inch represents $\frac{2000\times 10}{4\pi}$

$$=1600\text{ ergs per c.c. per cycle.}$$

Loss in the Iron per c.c. per cycle

$$=1600\times 1{\cdot}75$$

$$=2800\text{ ergs. } \textit{Ans.}$$

Example 21. The volume of iron in a transformer core built of sheet steel laminations is 6000 c.c The maximum flux density in the core is 10,000 lines per sq. cm. and the frequency is 50 cycles. Find the hysteresis loss in ergs per c.c. per cycle and the total power loss in watts.

Take the value of hysteresis coefficient as ·001.

Hysteresis loss per c.c. per cycle

$$={\cdot}001\times 10000^{1{\cdot}6}={\cdot}001\times 25{\cdot}12\times 10^5$$

$$=2512\text{ ergs.}$$

Total power loss in watts

$$=\frac{2512\times 6000\times 50}{10^7}$$

$$=75{\cdot}36\text{ watts. } \textit{Ans.}$$

Example 22. If the hysteresis loss in ergs per cycle per c.c. for 4% silicon steel is $10^{-3}\,B_{max}^{1.6}$, at what rate will the temperature of a core made of this material rise when the max. flux density is 10,000 lines per sq. cm. and frequency 50 cycles per sec., the density of steel is 7·5 and sp. heat ·114. The temp. rise may be expressed in degrees centigrade per hour assuming that there is no loss by radiation etc.

Let us assume 1000 c.c. of steel

Hysteresis loss in an hour

$$=10^{-3}\times 10000^{1.6}\times 1000\times 60\times 60\times 50$$

$$=25{\cdot}12\times 10^{5}\times\frac{1}{10^{3}}\times 10^{3}\times 18\times 10^{4} \text{ ergs.}$$

$$=45{\cdot}2\times 10^{10} \text{ ergs.}$$

$$=45{\cdot}2\times 10^{3} \text{ Joules.}$$

$$=10{\cdot}8\times 10^{3} \text{ calories.}$$

If t=temp. rise per hour we have :—

$$7500\times{\cdot}114\times t=10{\cdot}8\times 10^{3}$$

$$t=\frac{10{\cdot}8\times 10^{3}}{7500\times{\cdot}114}=12{\cdot}6^{\circ}\text{C}. \quad \textit{Ans.}$$

CHAPTER VIII

CHEMICAL EFFECTS OF CURRENT

8·1. Faraday's two laws of electrolysis are :

(*a*) The amount of an iron liberated and the electrolyte decomposed by the passage of electric current is directly proportional to the quantity of electricity which passes through the electrolyte. Thus if w is the weight of the ion liberated in time t ses. when Q quantity of electricity passes

w is proportional to Q or $I \times t$

or $w = ZIt$,

where I is the current and Z is called the **Electro-chemical equivalent** of the substance (ion).

The weight of an ion liberated by unit quantity of electricity is thus the Electro-chemical equivalent. This is a weight.

(*b*) If the same current flows through several electrolytes the weights of the ions liberated are proportional to their chemical equivalents.

$$\text{Chemical equivalent} = \frac{\text{Atomic weight}}{\text{Valency}}$$

This is a ratio,

In H_2O the valency of Oxygen is 2 and so its Chemical

$$\text{equivalent} = \frac{16}{2} = 8$$

Electro-chemical equivalent (Z) of a substance
= Z of Hydrogen × chemical equivalent of the substance.

Z of Hydrogen = ·0000104 gram per coulomb.

8·2. Current efficiency. In the process of electrolysis the quantity of a substance liberated is usually slightly less than the quantity calculated from the Faraday's laws. This is due to impurities etc. and is accounted for by a factor known as current efficiency.

$$\text{Current efficiency} = \frac{\text{Actual weight of substance liberated}}{\text{Calculated weight}}$$

It generally lies between 90 and 98%.

8·3. The chief practical applications of electrolysis are the extraction of metals from their ores, the refining of metals, the manufacture of various chemicals and electroplating, although there are other miscellaneous applications.

8·4. Chemical equivalents and Electro-chemical equivalents of some elements are given in the table below :

Element	*Atomic weight*	*Valency*	*Chemical Equivalent*	*Electro-Chemical Equivalent mg. per coulomb*
Aluminium	27·3	3	9·1	·0945
Cadmium	112·4	2	56·2	·582
Chromium	51·7	6	8·62	·090
Calcium	40·18	2	20 09	·2075
Chlorine	35 18	1	35·18	·3673
Copper (cupric)	63·1	2	31 55	·329
Gold	195·7	3	65·23	·6768
Hydrogen	1	1	1	·0104
Iron (Ferrous)	55 5	2	27·75	·2890
Lead	205·35	2	102·67	1·0716
Mercury (ous)	198 5	1	198·5	2 0719
Magnesium	23·94	2	11·97	·1243
Nickel	58 3	2	29·15	·3043
Nitrogen	14·00	3	4·67	·0485
Oxygen	15·88	2	7·94	·0829
Platinum	193 3	6	32·22	·3303
Silver	107·11	1	107·11	1·1180
Sodium	22·88	1	22·88	·2387
Sulphur	31 82	2	15·91	·1661
Sulphur		4	7·95	·0831
Sulphur		6	5·30	·0553
Tin (ous)	118·1	2	59 05	·6164
Zinc	64·9	2	32·45	·3387

Element liberated by 1000 ampere-hours

$=$Milligrams per coulomb $\times 7{\cdot}935$ lbs.

Thus copper liberated by 1000 amp-hours

$= {\cdot}329 \times 7{\cdot}935 = 2{\cdot}61$ lbs.

Example 1. How many coulombs and how many ampere-hours will be required to deposit $\frac{1}{4}$ lb. of silver from a solution of silver nitrate. What steady current in amps. will be required if the wok is to be done in 4 hours. Take Electro-chemical equivalent for silver$={\cdot}001118$.

$$\frac{1}{4}\text{lb.}=\frac{453\cdot6}{4}=113\cdot4 \text{ grams.}$$

Coulombs needed $=\dfrac{113\cdot4}{\cdot001118}=101{,}431$ coulombs

Amp.-hours $=\dfrac{101431}{3600}=28\cdot2$ amp.-hrs.

Current required $=\dfrac{28\cdot2}{4}=7\cdot05$ amps.

Example 2. What current will be required to decompose 1/10 litre of water in 10 hours. What volume of Hydrogen and Oxygen would be produced.

One litre $=1000$ c.c.

1/10 litre $=100$ c.c. $=100$ grams.

Wt. of Hydrogen set free

$$=\frac{100}{9}=11\cdot1 \text{ grams.}$$

If I is the current required then

$I\times\cdot0000104\times10\times60\times60=11\cdot1$

$$I=\frac{11\cdot1}{\cdot0000104\times10\times60\times60}$$

$=29\cdot7$ amps.

One c.c. of Hydrogen weighs $\cdot00008988$ gram

Vol. of Hydrogen $=\dfrac{11\cdot1}{\cdot00008988}$

$=123{,}621$ c.c.

Vol. of Oxygen is half the volume of Hydrogen.

Vol. of Oxygen $=61{,}810$ c.c. *Ans.*

Example 3. Taking the electro-chemical equivalent of silver as $\cdot001118$, calculate the current required to deposit a coating of silver $\cdot1$ m.m. thick on a surface of 1000 sq. cm. in one hour.

Density of silver $=10\cdot5$

Vol. of silver deposit $=1000\times\cdot01=10$ c.c.

Wt. of silver deposited $=105$ grams.

Current required $=\dfrac{105}{\cdot 001118\times 60\times 60}$

$=26$ amps. *Ans.*

Example 4. Determine the weight of zinc used in a Daniell cell in one hour if the total resistance of the circuit is 7 ohms.

Emf. of the Daniell cell

$=1\cdot 1$ Volts.

Current flowing $=\dfrac{1\cdot 1}{7}$ amp.

Atomic weight of zinc is 64·9 and Valency$=2$

Chemical equivalent $=32\cdot 45$

Electro-chemical equivalent

$=\cdot 000337$

Wt. of zinc used in one hour

$=\dfrac{1\cdot 1}{7}\times\cdot 000337\times 3600$

$=\cdot 191$ gram. *Ans.*

Example 5. How much caustic soda is produced per 100 amp.-hours and how much Lead and Mercury (from mercurous nitrate). One coulomb evolves ·0104 mg. of Hydrogen and the atomic weights of Sodium, Lead and Mercury are respectively 23, 207, 200.

Valency of Sodium $=1$, Ch. equivalent$=23$

,, ,, Lead $=2$, ,, ,, $=103\cdot 5$

,, ,, Mercury $=1$, ,, ,, $=200$

One coulomb liberates ·0104 mg. of Hydrogen

100 amp.-hours liberate Hydrogen

$=\dfrac{\cdot 0104\times 100\times 3600}{1000}=3\cdot 744$ grams.

Sodium liberated $=3\cdot 744\times 23=86\cdot 1$ grams.

Lead ,, $=3\cdot 744\times 103\cdot 5=387\cdot 5$ grams.

Mercury ,, $=3\cdot 744\times 200=748\cdot 8$ grams.

23 grams of sodium produces 40 grams of NaOH.

86·1 grams of sodium will give

$$=\frac{40}{23}\times 86{\cdot}1$$

$$=149{\cdot}7 \text{ grams of NaOH.} \quad \textit{Ans.}$$

Example 6. In a copper refining plant 250 cells are connected in series and carry a current of 4000 amps. The voltage per cell is ·25 V. If the plant works 80 hours a week find the annual output of copper and energy used in Kwh per ton.

Electro-chemical equivalent of copper

$$={\cdot}329 \text{ mg. per coulomb}$$

E.C.E in lbs. per 1000 amp.-hours

$$={\cdot}329\times\frac{1}{1000}\times\frac{1}{453{\cdot}6}\times 1000\times 3600$$

$$={\cdot}329\times 7{\cdot}935=2{\cdot}61 \text{ lbs. per 1000 AH}$$

Time $=52\times 80=4160$ hours

Wt. of copper per cell per year

$$=\frac{2{\cdot}61}{1000}\times 4000\times 4160 \text{ lbs.}$$

Total out-put $=\dfrac{2{\cdot}61\times 4\times 4160\times 250}{2240}$ tons.

$$=4847 \text{ tons.} \quad \textit{Ans.}$$

Voltage across 250 cells

$$=250\times{\cdot}25=62{\cdot}5 \text{ V.}$$

Total energy consumed per year

$$=\frac{62{\cdot}5\times 4000}{1000}\times 4160$$

$$=1{,}040{,}000 \text{ units}$$

Energy used per ton $=\dfrac{1040000}{4847}=214$ kwh.

Example 7. How much aluminium would be produced from aluminium oxide in a working week of 100 hours in a cell carrying an average current of 5000 amps. The current efficiency is 70%. If the voltage of the cell is 6 V. and energy

costs $\frac{1}{4}$ *d*. per unit what would be the energy cost per ton of aluminium produced. Electro-chemical equivalent of aluminium is ·0945 mg. per coulomb.

E.C.E. per 1000 amp. hrs.

$$=\cdot0945\times7\cdot935 \text{ lbs.}$$

Wt. of Al. $$=\frac{\cdot0945\times7\cdot935\times5000\times100\times\cdot7}{1000}$$

$$=262\cdot5 \text{ lbs.}$$

Cost of energy $$=\frac{5000\times6}{1000}\times100\times\frac{1}{4}\times\frac{1}{12}\times\frac{1}{20}$$

$$=\pounds\ 3\cdot125$$

Energy cost per ton. $$=\frac{2240}{262\cdot5}\times3\cdot125=\pounds26\cdot6.$$

Example 8. Find the value of the steady current required to decompose 10 grams of water in one hour.

The electro-chemical equivalent Z to be used in this case is the sum of the Electro-chemical equivalent for hydrogen and oxygen

$$=\cdot0104+\cdot829$$

$$=\cdot0933 \text{ mg. per coulomb.}$$

$$I\times\cdot0933\times3600=10\times1000$$

$$I=\frac{10\times1000}{\cdot0933\times3600}=29\cdot7 \text{ amps.}\quad \textit{Ans.}$$

CHAPTER IX

THE D.C. GENERATOR

9-1. The magnitude of emf. induced in a conductor when moved at right angles to a magnetic field is

$$e = Blv \times 10^{-8} \text{ volts.} \qquad \text{...Eq. (9-1)}$$

where B = lines of force/sq. cm.

l = length of conductor in cm.

v = velocity in cms/sec.

or

$$e = \frac{\text{Lines of force cut per sec}}{10^8} \text{ volts.}$$

The direction of emf. thus induced is easily found with the help of Fleming's Right Hand rule.

9-2. The equation for the emf. generated in D.C. generator is :—

$$E = \frac{p\phi zN}{10^8 \times q \times 60} \qquad \text{...Eq. (9-2)}$$

where E = Induced emf. in volts

p = Number of poles

ϕ = Flux per pole

z = Number of armature conductors.

N = Speed in R.P.M.

q = Number of armature circuits.

Note. Sometimes "p" is taken to represent the number of pairs of poles and "a" is used to represent the number of armature circuits. These should not cause any confusion and the emf. equation can be changed accordingly.

In some cases the useful magnetic flux has to be calculated to find the generated emf. The following points should be carefully noted.

(a) Flux = Flux density × area.

(*b*) The flux passing through the poles and yoke is greater than the flux passing through the air gap. This is accounted by a factor called the leakage factor and

$$\text{Flux in the air gap} = \frac{\text{Flux in the pole}}{\text{Leakage factor}}.$$

The value of this leakage factor lies generally between 1·1 and 1·2.

(*c*) Assuming that there is no leakage, the flux passing through each section of the armature core or the yoke is half

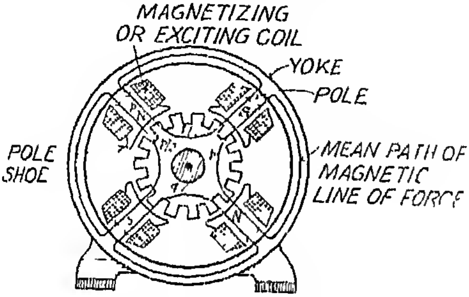


Fig 58. Magnetic field distribution in the Dynamo.

the value of flux in the pole or air gap. This is illustrated in figure 58.

9·3. Classification of Dynamos.

The D.C. generators are classified according to the method of their field excitation. The various types are:—

(*a*) **Separately excited Dynamo.** The field magnets are

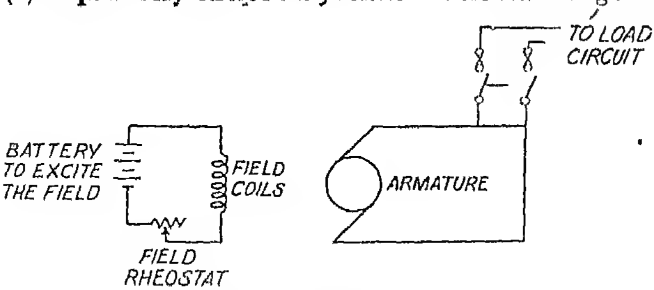


Fig 59. Separately excited Dynamo

excited from an entirely independent source of D C. supply.

(*b*) **Shunt Dynamo.** The field coils are wound with many turns of thin wire and connected across the armature.

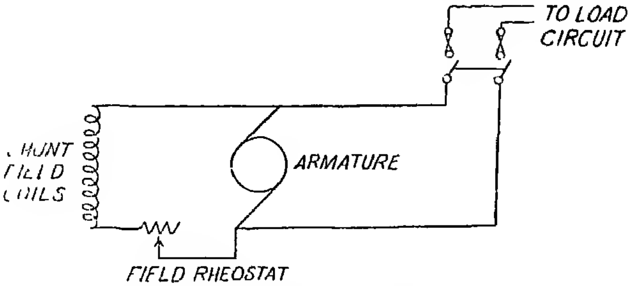


Fig. 60. Shunt Dynamo

(*c*) **Series Dynamo.** The field coils are wound with a few turns of wire of heavy cross-sectional area and connected in series

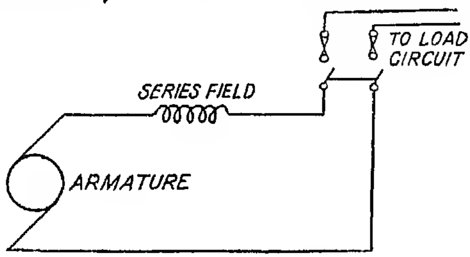


Fig. 61. Series Dynamo

with the armature The machine will excite only when the external load circuit is put on.

(*d*) **Short-shunt Compound Dynamo.** There are two sets of field coils, one set connected across the armature and other in series with the armature.

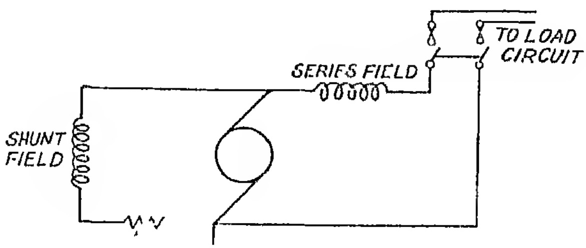


Fig. 62. Short-shunt Compound Dynamo

(e) **Long-shunt Compound Dynamo.** In this also there are two sets of field coils, one set is connected in series with the

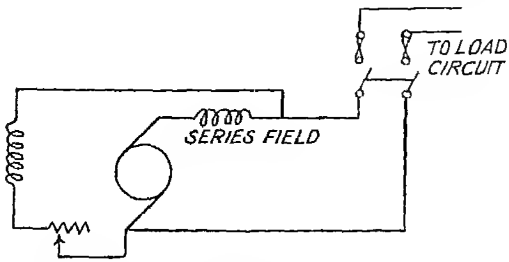


Fig. 63. Long-shunt Compound Dynamo

armature and other across the terminals of the armature and series field.

9-4. Lap and Wave Windings.

The subject of armature winding will be discussed in the next chapter but the way in which the type of winding affects the generated emf is explained below.

The two methods of connecting up the armature conductors are (a) Wave winding or Two circuit winding, (b) Lap winding or Multiple circuit winding.

(a) **Wave winding.** The armature conductors are connected in two parallel paths irrespective of the number of poles, and each path supplies half of the total armature current. Two sets of brushes only are necessary but it is usual to provide as many sets of brushes as there are poles. The majority of low output machines have wave wound armatures. For wave wound machine

$$q=2 \qquad \text{...Eq. (9-3)}$$

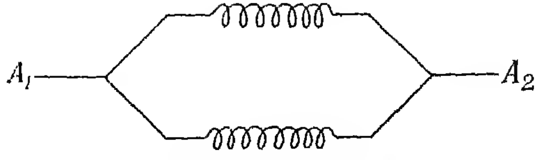


Fig. 64. Two circuits in a wave wound armature

(*b*) **Lap Winding.** The armature has as many parallel circuits and sets of brushes as the number of poles. Therefore for lap wound machine

$$q=p \qquad \text{...Eq. (9-4)}$$

The total armature current divides equally among the parallel armature circuits. All machines of large output have an armature winding of this type

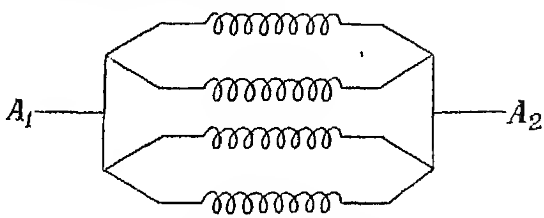


Fig. 65. 4 circuits of a 4 pole lap wound armature

Example 1. A conductor 12″ long rotates about one end at 1000 rpm. in a plane perpendicular to a magnetic field of strength 5000 lines per sq. cm. Find the emf. induced in it.

$$\text{Emf.}=\frac{\text{Lines of force cut per sec.}}{10^8}\text{ volts.}$$

Area cut per sec

$$=\pi\times12\times12\times2{\cdot}54\times2{\cdot}54\times\frac{1000}{60}\text{ sq. cms.}$$

$$\text{Emf. induced}=\pi\times144\times2{\cdot}54^2\times\frac{1000}{60}\times\frac{5000}{10^8}\text{ volts}$$

$$=2{\cdot}42\text{ volts. } \textit{Ans.}$$

Example 2. Find the emf. induced in a conductor 40 cms. long moving with a velocity of 30 metres per sec. at rt. angles to a uniform field of 8000 lines per sq. cm.

$$e=\mathrm{B}lv\times10^{-8}$$

$$\mathrm{B}=8000\text{ lines per sq. cm.}$$

$$l=40\text{ cms.}$$

$$v=30\times100\text{ cms. per sec.}$$

$$\text{Emf. induced }=\frac{8000\times40\times30\times100}{10^8}=9{\cdot}6\text{ volts. } \textit{Ans.}$$

Example 3. The active length of a conductor in the armature of a D.C. generator is 25 cms. The peripheral speed of the conductor is 2000 cms. per sec. and the flux density at

the centre of the poles is 9000 lines per sq. cm. Find the induced emf. in the conductor when moving under the centre of the pole.

$$e = Blv \times 10^{-8}$$

$$B = 9000 \text{ lines per sq. cm.}$$

$$l = 25 \text{ cms.}$$

$$v = 2000 \text{ cms. per sec.}$$

$$E = \frac{9000 \times 25 \times 2000}{10^8} = 4{\cdot}5 \text{ volts. } \textit{Ans.}$$

Example 4. The maximum emf. induced per conductor on a certain D.C. generator is 10 volts. The maximum air gap flux density is 10000 lines per sq. cm. Calculate the maximum possible active length of the conductor under these conditions when the peripheral speed is 25 metres per sec.

$$e = \frac{Blv}{10^8}$$

$$B = 10{,}000$$

$$l = ?$$

$$v = 2500 \text{ cms. per sec.}$$

$$10 = \frac{10000 \times l \times 2500}{10^8}$$

$$l = 40 \text{ cm. } \textit{Ans.}$$

Example 5. A shunt dynamo supplied a load of 20 KW at 200 volts through feeders having a resistance of ·1 ohm. The shunt field resistance is 100 ohms and the armature resistance ·105 ohm. Find the terminal p.d. of the machine and the generated emf.

Load current $= \frac{20 \times 1000}{200} = 100$ amps.

Voltage drop in the feeders

$= 100 \times {\cdot}1 = 10$ volts

P.d. across the machine

$= 200 + 10 = 210$ V.

Shunt field current $= \frac{210}{100} = 2{\cdot}1$ A.

Armature current $= 100 + 2{\cdot}1 = 102{\cdot}1$ amps.

Armature drop $= 102{\cdot}1 \times {\cdot}05 = 5{\cdot}1$ volts

Emf. generated $= 210 + 5{\cdot}1 = 215{\cdot}1$ volts. *Ans.*

Example 6. A short-shunt compound dynamo feeds a load taking 100 amps at 200 V. The resistance of the leads is ·1 ohm, armature resistance ·05 ohm, shunt field resistance 100 ohms and series field resistance ·015 ohm. Find the emf. generated.

Volts drop in the leads	$=100\times\cdot1=10$ volts.
P.d. across the dynamo terminals	$=210$ volts.
Volts drop in the series field	$=\cdot015\times100$
	$=1\cdot5$ volts.
P.d. across the armature	$=210+1\cdot5=211\cdot5$ V.
Shunt field current	$=\frac{211\cdot5}{100}=2\cdot115$A.
Armature current	$=100+2\cdot115=102\cdot115$ A.
Armature voltage drop	$=102\cdot115\times\cdot05=5\cdot1$ volts.
Emf. generated	$=211\cdot5+5\cdot1=216\cdot6$ volts.

Ans.

Example 7. A 4 pole shunt generator with a lap wound armature supplies a load of 200 amps at 100 volts. Shunt field resistance is 50 ohms, and armature resistance is ·05 ohm. Calculate

(*a*) Total armature current.

(*b*) Current per armature circuit.

(*c*) Emf. generated.

Allow a brush contact drop of 2 volts.

Shunt field current	$=\frac{100}{50}=2$ amps.
Armature current	$=200+2=202$ amps.

In a lap wound armature the number of armature circuits is the same as the number of poles.

No. of circuits	$=4$
Current per circuit	$=\frac{202}{4}=50\cdot5$ amps.
Terminal p.d.	$=100$ V.
Brush drop	$=2$ V.

Armature resistance drop $=202\times\cdot05=10\cdot1$ V.

Emf. generated $=100+2+10\cdot1$

$=112\cdot1$ volts. *Ans.*

Example 8. A 4 pole D.C. generator has a wave wound armature with 37 slots, each slot containing 24 conductors. The useful magnetic flux is ·6 mega-lines per pole. Find the emf. generated in the armature if it is running at 1500 rpm.

$$E=\frac{p\phi ZN}{10^8 q 60}$$

$p=4$

$\phi=\cdot6\times10^6$

$Z=37\times24$

$N=1500$

$q=2$

$$E=\frac{4\times\cdot6\times10^6\times37\times24\times1500}{10^8\times2\times60}=266\cdot4 \text{ volts. } \textit{Ans.}$$

Example 9. A 6 pole 1000 rpm. D. C. generator has a lap wound armature with 288 conductors. The poles are inches square and the average flux density in the air gap is 50,000 lines per sq. inch. Calculate the emf. generated by the machine.

$p=6$

$\phi=10\times10\times50{,}000=5\times10^6$ lines

$Z=288$

$N=1000$

$q=6$

$$E=\frac{6\times5\times10^6\times288\times1000}{10^8\times6\times60}=240 \text{ volts. } \textit{Ans.}$$

Example 10. A 6 pole generator has a wave wound armature. The axial length of the armature is 40 cms. and its diameter 30 cms. The poles cover $\frac{2}{3}$ of the circumference and the air gap density is 6000 lines per sq. cms. Find the number of armature conductors so that the induced emf. is 220V when the machine runs at 250 rpm.

Total area under the poles

$$=\pi\times30\times40\times\frac{2}{3}=2520 \text{ sq. cm.}$$

Total flux under the 6 poles

$$=p\phi=2520\times6000 \text{ lines}$$

$$220=\frac{2520\times6000\times Z\times250}{10^8\times2\times60}$$

$$Z=\frac{220\times10^8\times2\times60}{2520\times6000\times250}=700 \quad \textit{Ans.}$$

Example 11 The flux density in the pole core of an 8 pole generator is 15,000 lines per sq cm. The pole cores are 40 cm. in diameter and the leakage coefficient is ·12. There are 704 conductors on the armature and the machine is lap wound Find the emf. induced when the machine is driven at 300 rpm.

Total flux in each pole core

$$=\frac{\pi}{4}\times40\times40\times15000 \text{ lines}$$

Useful flux

$$=\frac{\pi}{4}\times\frac{40\times40}{1\cdot2}\times15000 \text{ lines}$$

$$=\pi\times400\times12500$$

Induced emf.

$$=\frac{8\times\pi\times400\times12500\times704\times300}{10^8\times8\times60}=553 \text{ volts.} \quad \textit{Ans.}$$

Example 12. The armature of a 4 pole shunt dynamo is lap wound and generates an emf of 200 volts when driven at 600 rpm. The armature has 864 conductors. It is desired that the machine should develop 450 V. by reconnecting the armature wave and changing its speed. Find the speed to produce the required emf.

$$E=\frac{p\phi ZN}{10^8 q 60}$$

In this p, ϕ, Z, 10^8, 60 are constants The equations can be written as :—

$$E = \frac{KN}{q}$$

$$200 = \frac{K \times 600}{4}$$

$$K = \frac{200 \times 4}{600} = \frac{4}{3}$$

When the machine develops 450 V.

$$450 = \frac{4}{3} \times \frac{N}{2}$$

$$N = \frac{450 \times 3 \times 2}{4} = 450 \times 1 \cdot 5$$

$$= 675 \text{ rpm.} \quad \textit{Ans.}$$

Example 13. A workshop manufacturing 220V. D.C. fans needs 220 V. D.C. supply for testing purposes. It has a

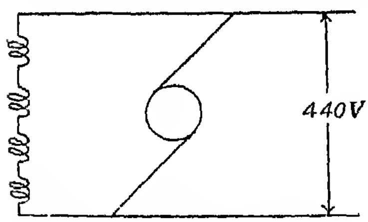


Fig. 66. Original 440 V. machine 2000 rpm 4 shunt field coils connected as shown

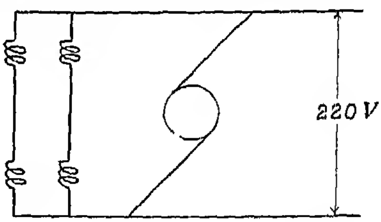


Fig. 67. Machine connected to produce 220V at 1000 rpm. The shunt field coils are connected in pairs across the armature

4 pole 440 V., 2000 rpm. shunt generator. Suggest the cheapest method so that the machine in hand may be used for testing purposes.

$$E = \frac{p\phi ZN}{10^8 q 60}$$

p, Z, q are all constants if the armature winding is not disturbed.

If the flux could be kept constant and the machine driven at 1000 rpm. it will generate 220 V.

In Fig. 66 the shunt field winding is distributed in 4 coils connected in series and the combination connected across the armature. Each shunt field coil receives 110 volts The excitation current is $\frac{110}{R}$ where R is the resistance of one coil of shunt winding. If the shunt field coils are re-arranged as in Fig. 67, and the machine run at 1000 rpm. the excitation is again $\frac{110}{R}$ *i.e.*, flux remains constant.

Hence our purpose is served.

CHAPTER X

ARMATURE WINDING

10-1. A simple coil consists of two conductors. One of these conductors is placed at the top of a slot and the other conductor placed at the bottom of another slot approximately a pole pitch apart. One side of the coil is called Coil Side, Leg or Half Coil. In the case of a simple coil stated above each coil side consists of only one conductor. The two ends of the simple coil are connected to :—

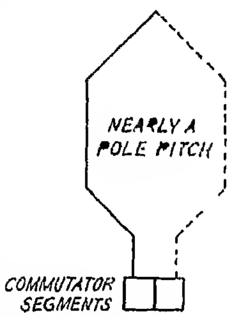


Fig. 68. Simple coil of a lap wound machine.

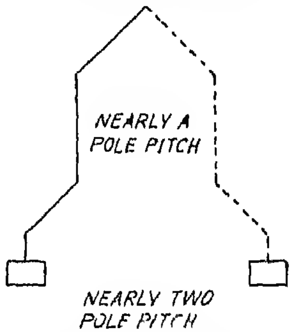


Fig. 69. Simple coil of a wave wound machine.

(*a*) Adjacent commutator segments or bars in the case of a lap winding (Fig. 68.)

(*b*) Commutator bars nearly two pole pitch apart in the case of a wave winding. (Fig. 69.)

In the construction of winding diagrams the coil side at the top of the slot is generally shown by full line and that at the bottom of the slot by dotted line. Or the top side can be shown in red and the bottom in blue.

A coil may consist of more than two conductors and each coil side may have 2, 3 or more conductors. A coil with 4 conductors or two conductors per coil side is shown in Fig. 70.

10-2. Definitions.

Coil span is the distance stepped forward in connecting one side of a coil to the other side and may conveniently be

stated in terms of the number of armature teeth embraced by the coil. The coil span is approximately a pole pitch *i.e.*, the distance between the pole centres.

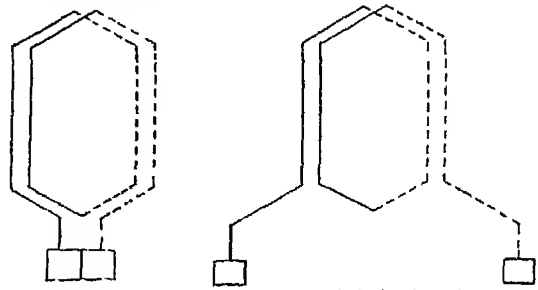


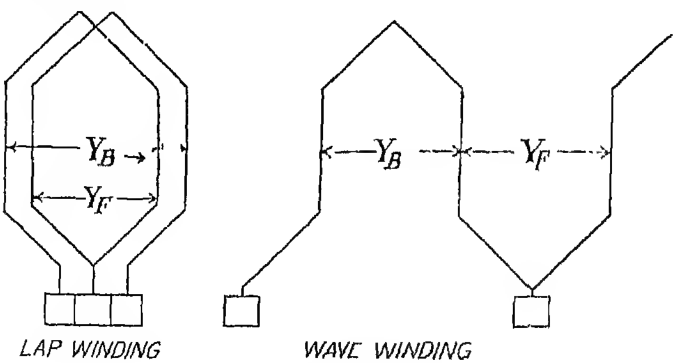
Fig 70 Coils with 2 turns or 4 conductors per coil.

If the coil span is exactly a pole pitch the coil is said to be a *Full pitch* coil. If the coil span is less than a pole pitch it is said to be a *Short pitch* or chorded coil.

Commutator pitch is the distance between the two segments connected directly to the two ends of a coil.

Back pitch Y_B. It is the distance between the two coil sides of a coil.

Front pitch. It is the distance between the two ends of



Y_B is Back pitch
Y_F is Front pitch

Fig. 71. Back and Front Pitches.

two different coils which are connected to the same commutator segment

10-3. Information about D.C. windings and winding diagrams.

(*i*) The commutator side of the armature is called the front side and the other one as the back side.

(*ii*) In numbering of the coil-sides or legs, the coil sides at the top of the slots are numbered 1, 3, 5, 7 etc. The coil sides at the bottom of the slots are numbered 2, 4, 6, 8, 10 etc. Thus with simple coils each coil having 2 conductors and with 2 coil sides per slot or with two conductors per slot :

Slot No. 1 has coil sides 1 and 2
Slot No. 2 ,, ,, ,, 3 and 4
Slot No. 3 ,, ,, ,, 5 and 6
and so on.

(*iii*) The back and front pitches are both odd, in both Lap and Wave windings. Thus if the back pitch is 17 and front pitch is 15 in the case of a lap winding, the coil side No. 1 in slot No. 1 is connected to coil side No. 18 in slot No. 9. Then coil side No. 18 and coil side No. 3 in slot No. 2 are connected to the same commutator segment, and so on.

(*iv*) Number of commutator segments

=Number of coils.

(*v*) Number of conductors on the armature

=Number of coils $\times$ turns per coil $\times$ 2.

(*vi*) Number of conductors per slot

=coils per slot $\times$ turns per coil $\times$ 2.

Example 1. Draw up the winding table for a 4 pole 16 slot lap connected armature with one coil per slot and one turn per coil. Draw a developed diagram of the armature winding showing the position and polarity of the brushes, the direction of the current in the various conductors, the short circuited coils, the position of the poles and the direction of rotation for a generator.

Show also that there are 4 parallel paths through the armature.

There are 4 slots per pole and for a full pitch winding the top conductor (conductor No. 1 or coil side No. 1) should

be connected to the conductor at the bottom of slot No. 5 (coil side No. 10 which will be connected to coil side No. 3 at the front). The back pitch, therefore, is 9 and the front pitch 7. (Observe in lap winding $y_B - y_F = 2$).

Winding Table

Back Connections	*Front Connections*
1 to 10	10 to 3
3 to 12	12 to 5
5 to 14	14 to 7
7 to 16	16 to 9
9 to 18	18 to 11
11 to 20	20 to 13
13 to 22	22 to 15
15 to 24	24 to 17
17 to 26	26 to 19
19 to 28	28 to 21
21 to 30	30 to 23
23 to 32	32 to 25
25 to 2	2 to 27
27 to 4	4 to 29
29 to 6	6 to 31
31 to 8	8 to 1

Draw the winding diagram as shown. Number the coil sides. Put the brushes at equal distances on the commutator.

Negative brush I short circuits commutator bars 1 and 2 and coil sides No. 1, 10.

Negative brush II short circuits commutator bars 9 and 10 and coil sides No. 17, 26.

Positive brush I short circuits commutator bars 5 and 6 and short circuits coil sides No. 9, 18.

Positive brush II short circuits commutator bars 13 and 14 and short circuits coil sides No. 25, 2.

To show that there are 4 parallel circuits through the armature :—

Starting from negative brush I, via commutator segment No. 2 the current passes through coil sides 3, 12, 5, 14, 7, 16 to positive brush I.

Again from negative brush I via segment No. 1 the current passes through coil sides 8, 31, 6, 29, 4, 27 to positive brush II.

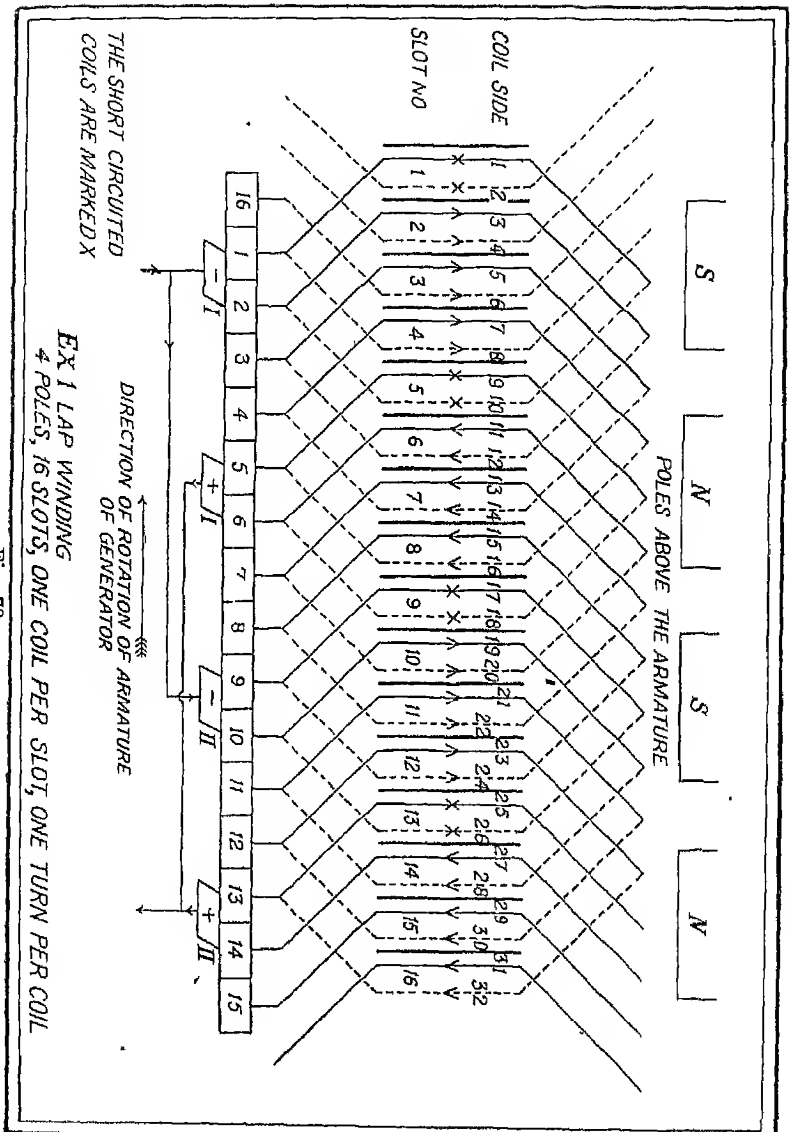

POLES ABOVE THE ARMATURE
S
N
S
N
COIL SIDE
SLOT NO
DIRECTION OF ROTATION OF ARMATURE OF GENERATOR
THE SHORT CIRCUITED COILS ARE MARKED X
EX 1 LAP WINDING
4 POLES, 16 SLOTS, ONE COIL PER SLOT, ONE TURN PER COIL

Fig. 72

Similarly there are two circuits from negative brush II—one to positive brush I and other to positive brush II.

Thus there are 4 parallel circuits through the armature.

Position of the Poles

The short-circuited coil sides 1, 2, 9, 10 must lie in the neutral zone. The pole pitch extends from centre of coil sides 1 and 2 on the diagram to centre of coil sides 9 and 10. The pole arc is generally ·7 of the pole pitch and thus all the poles are drawn in the diagram.

Direction of Rotation of Armature.

The poles are above the armature in the diagram and by the application of Fleming's Right Hand Rule the direction of rotation of the armature is obtained as shown.

In this example :

Coils per slot $=1$

Turns per coil $=1$

Conductors per slot $=$coils per slot $\times$ turns per coil $\times 2$

$=1\times1\times2=2$

Total number of coils $=16$

Number of commutator bars

$=$No. of coils$=16$

Number of coil sides $=$Coils$\times2=16\times2=32$

Number of armature conductors

$=$No. of coils $\times$ turns per coil $\times 2$

$=16\times1\times2=32$

Commutator pitch $=2-1=1$.

Example 2. Draw up the winding table for a 4 pole 16 slot lap connected armature with 2 coils per slot and one turn per coil. Draw a developed diagram of the armature winding showing the position and polarity of the brushes, the direction of the currents in the various conductors, the short-circuited coils, the position of the poles and the direction of rotation of the armature for a generator.

Here, as in Example 1, there are 4 slots per pole but there are two coils per slot or 4 coil-sides per slot. For a full pitch winding the coil side No. 1 in slot No. 1 shall be connected to

the symmetrically placed coil side at the bottom of slot No. 5. This coil side number is 18.

Therefore back pitch $=18-1=17$

Front pitch $=17-2=15$

Total No. of coil sides $=16\times4=64$.

Winding Table

Back Connection	*Front Connection*
1 to 18	18 to 3
3 to 20	20 to 5
5 to 22	22 to 7
7 to 24	24 to 9
9 to 26	26 to 11
11 to 28	28 to 13
13 to 30	30 to 15
15 to 32	32 to 17
17 to 34	34 to 19
19 to 36	36 to 21
21 to 38	38 to 23
23 to 40	40 to 25
25 to 42	42 to 27
27 to 44	44 to 29
29 to 46	46 to 31
31 to 48	48 to 33
33 to 50	50 to 35
35 to 52	52 to 37
37 to 54	54 to 39
39 to 56	56 to 41
41 to 58	58 to 43
43 to 60	60 to 45
45 to 62	62 to 47
47 to 64	64 to 49
49 to 2	2 to 51
51 to 4	4 to 53
53 to 6	6 to 55
55 to 8	8 to 57
57 to 10	10 to 59
59 to 12	12 to 61
61 to 14	14 to 63
63 to 16	16 to 1

Draw the winding diagram as shown in fig 73. Number the coil sides.

The number of commutator segments here is 32. Put the brushes at equal distances on the commutator.

Negative brush I short-circuits commutator bars 1 and 2 and coil-sides 1 and 18.

Negative brush II short-circuits commutator bars 17 and 18 and coil sides 33 and 50.

Positive brush I short circuits commutator bars 9 and 10 and coil sides 17 and 34.

Positive brush II short circuits commutator bars 25 and 26 and coil sides 49 and 2.

The position of the poles and the direction of rotation is obtained as explained in Example 1.

In this example :

Coils per slot $=2$

Turns per coil $=1$

Conductors per slot $=$ Coils per slot $\times$ turns per coil $\times 2$

$=2\times1\times2=4$

Total number of coils $=32$

No. of commutator bars $=$ No. of coils $=32$

No. of coil-sides $=32\times2=64$

No. of armature conductors

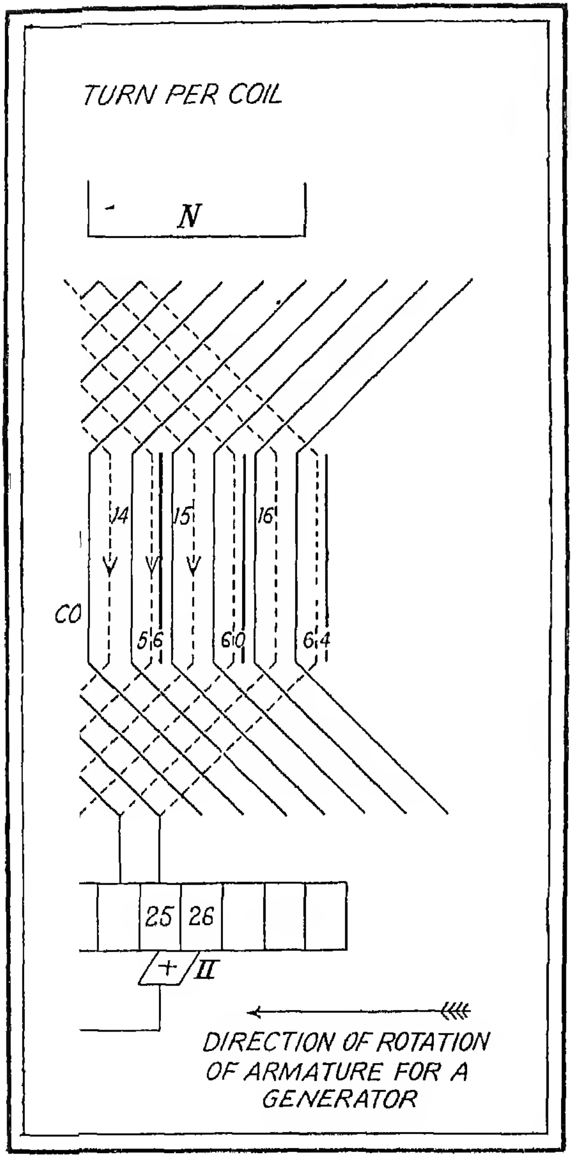
$=$ No. of coils $\times$ turns per coil $\times 2$

$=32\times1\times2=64$

Commutator pitch $=2-1=1$. *Ans.*

Example 3. Draw up the winding table for a 4 pole 17 slot progressive wave connected armature with one coil per slot and one turn per coil. Draw a developed diagram of the armature winding showing the position and polarity of brushes, the direction of the current in the various conductors, the short-circuited coils, the position of the poles and the direction of rotation for a generator.

Show also that there are two parallel paths through the armature.



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Number of coils $=17$

Number of coil sides $=17\times2=34$

In the wave winding, the pitch

$$=\frac{\text{No. of coil sides}\pm2}{\text{poles}}$$

$$=\frac{34\pm2}{4}=\frac{36}{4}\text{ or }\frac{32}{4}$$

$$=9 \text{ or } 8.$$

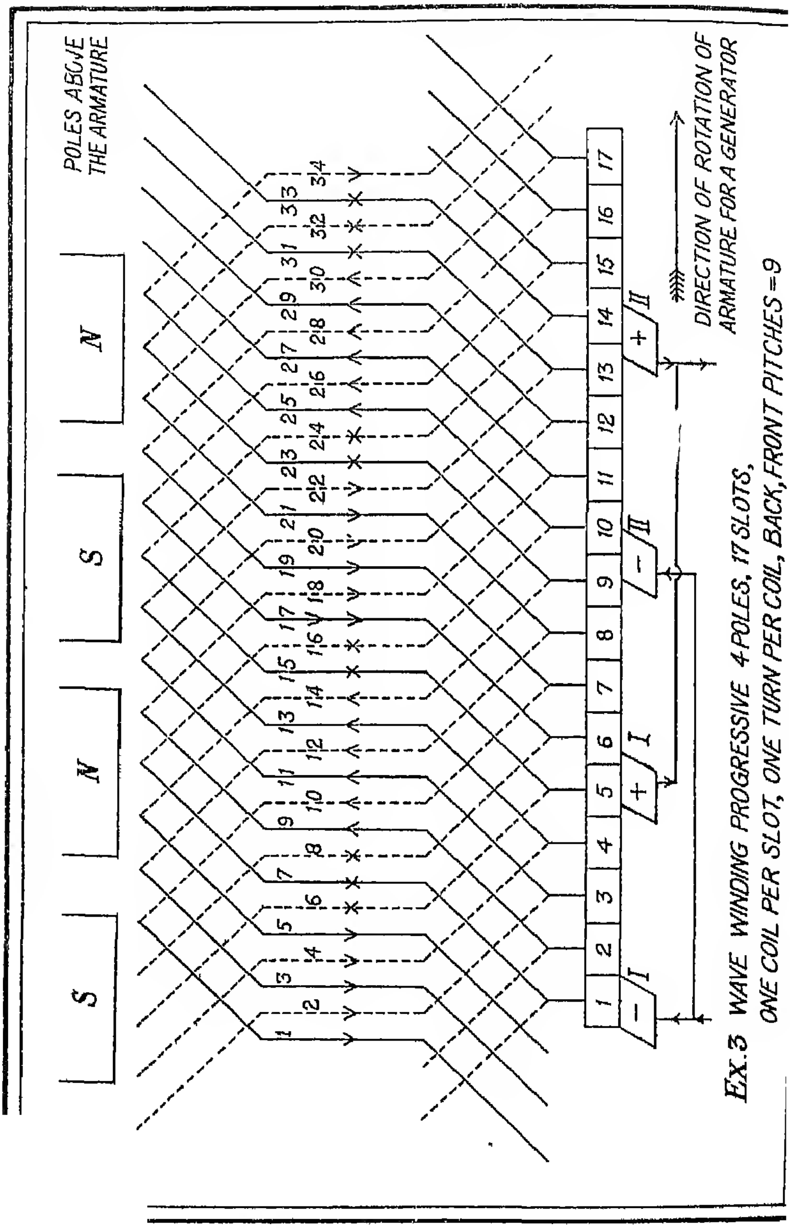
The pitch 9 gives a progressive winding and the pitch 8 gives a retrogressive winding as explained in Example 4.

Winding Table

Back Connection	*Front Connection*
1 to 10	10 to 19
19 to 28	28 to 3
3 to 12	12 to 21
21 to 30	30 to 5
5 to 14	14 to 23
23 to 32	32 to 7
7 to 16	16 to 25
25 to 34	34 to 9
9 to 18	18 to 27
27 to 2	2 to 11
11 to 20	20 to 29
29 to 4	4 to 13
13 to 22	22 to 31
31 to 6	6 to 15
15 to 24	24 to 33
33 to 8	8 to 17
17 to 26	26 to 1

Draw the winding diagram as shown in Fig. 74. Number the coil sides. Put the brushes at equal distances on the commutator. There being 17 coils there are 17 commutator segments and the distance from the centre of one brush to the next is $4\frac{1}{4}$ commutator bars. From the winding diagram we see that:

Negative brushes I; II short-circuit coil sides 7, 16, 23 and 32.



Ex. 3 WAVE WINDING PROGRESSIVE 4 POLES, 17 SLOTS, ONE COIL PER SLOT, ONE TURN PER COIL, BACK, FRONT PITCHES = 9

Positive brushes I, II short-circuit coil sides 15, 24, 33, 8, 31 and 6.

To show that there are two parallel paths through the armature :—

Starting from negative brush II there are two parallel paths through the armature—one going to positive brush I and other to positive brush II. At the instant shown there is no current flow from negative brush I to the armature. The circuits are :—

1. Negative brush II to coil-sides 14, 5, 30, 21, 12, 3, 28, 19, 10, 1, 26, 17 to positive brush I.
2. Negative brush II to coil sides 25, 34, 9, 18, 27, 2, 11, 20, 29, 4, 13, 22 to positive brush II.

Position of poles.

The short circuited coil sides are 6, 7, 8 then 15, 16, then 23, 24 and then 31, 32, 33.

The neutral axis in the diagram is on coil side 7, at middle of coil sides 15, 16 ; at middle of coil sides 23, 24 and on coil side 32. The poles can be now marked as ·7 of distance from one neutral axis to other.

Direction of rotation.

The poles are above the armature and the direction of rotation is found by the Fleming's right hand rule for generator.

Commutator pitch.

Coil No 1 with coil sides 1 and 10 is connected to commutator bars 1 and 10.

Therefore commutator pitch $=10-1=9$

$=$ Mean winding pitch.

Example 4. With the data given in example 3 make a retrogressive winding giving also the winding table etc.

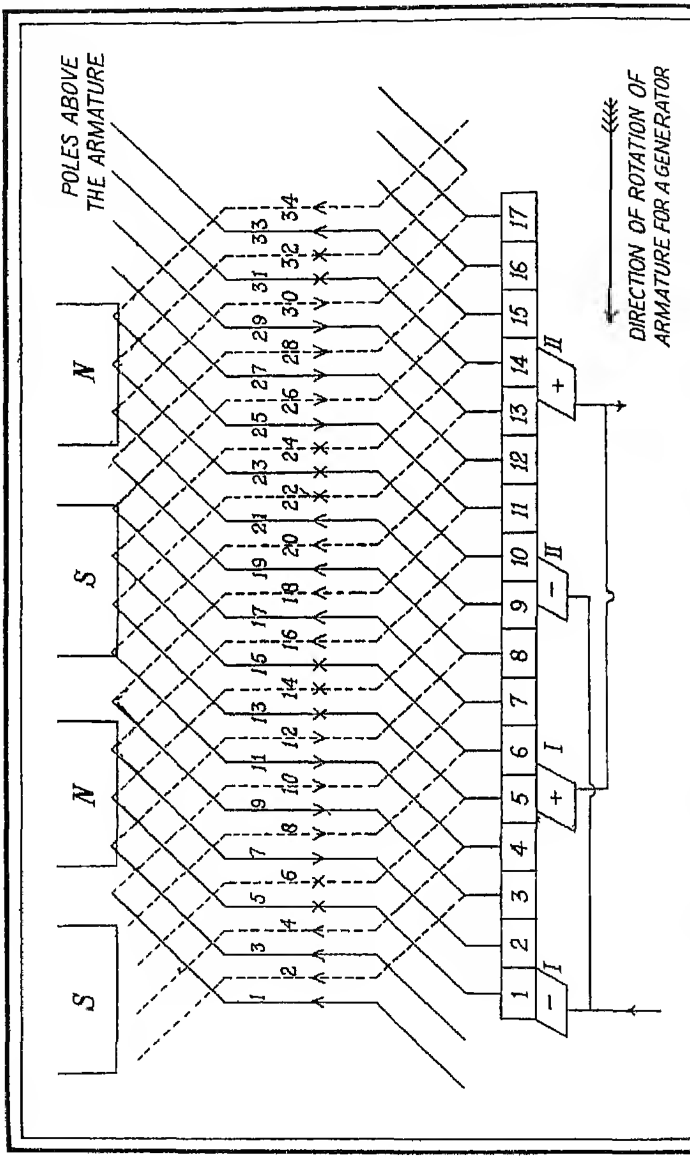
As stated in example 3 the total No. of coil sides $=34$

The pitch 8 gives a retrogressive winding. The pitches should be odd and not even, therefore we make :—

Back pitch $=9$

Front pitch $=7$

Making the mean pitch $=\frac{9+7}{2}=8$.



Ex. 4 WAVE WINDING RETROGRESSIVE. 4 POLES, 17 SLOTS, ONE COIL PER SLOT, ONE TURN PER COIL, BACK PITCH 9, FRONT PITCH 7.

Winding Table

Back Connection	*Front Connection*
1 to 10	10 to 17
17 to 26	26 to 33
33 to 8	8 to 15
15 to 24	24 to 31
31 to 6	6 to 13
13 to 22	22 to 29
29 to 4	4 to 11
11 to 20	20 to 27
27 to 2	2 to 9
9 to 18	18 to 25
25 to 34	34 to 7
7 to 16	16 to 23
23 to 32	32 to 5
5 to 14	14 to 21
21 to 30	30 to 3
3 to 12	12 to 19
19 to 28	28 to 1

Draw the winding diagram as shown in fig 75. Number the coil sides and put the brushes as in example 3. From the diagram we see that :

Negative brushes I, II short-circuit the coil sides 5, 14, 23 and 32.

Positive brushes I, II short-circuit the coil sides 13, 22, 15, 24, 31 and 6.

The position of the poles and the direction of rotation are found out as already explained in example 3. In this winding :

Coils per slot $=1$

Turns per coil $=1$

Conductors per slot $=$ coils per slot $\times$ turns per coil $\times 2$

$=1\times1\times2=2$

Total number of coils $=17$

Number of commutator bars $=17$

Total number of armature conductors

$$= \text{Number of coils} \times \text{turns per coil} \times 2$$

$$= 17 \times 1 \times 2 = 34$$

Commutator pitch $=$ mean winding pitch $= 8$ *Ans.*

Example 5. Draw up a part of the winding table for a 4 pole 17 slot wave connected armature with 3 coils per slot and one turn per coil. Draw the developed winding diagram, show the position of the brushes, the direction of currents in the various conductors, the poles and the direction of rotation for a generator.

In this example

Coils per slot $= 3$

Turns per coil $= 1$

Conductors per slot $= 3 \times 1 \times 2 = 6$

Total number of conductors

$$= \text{No. of coils} \times \text{turns per coil} \times 2$$

$$= \text{No. of slots} \times \text{coils per slot} \times \text{turns per coil} \times 2$$

$$= 17 \times 3 \times 1 \times 2 = 10$$

No. of coils $= 17 \times 3 = 51$

No. of commutator bars

$$= 51$$

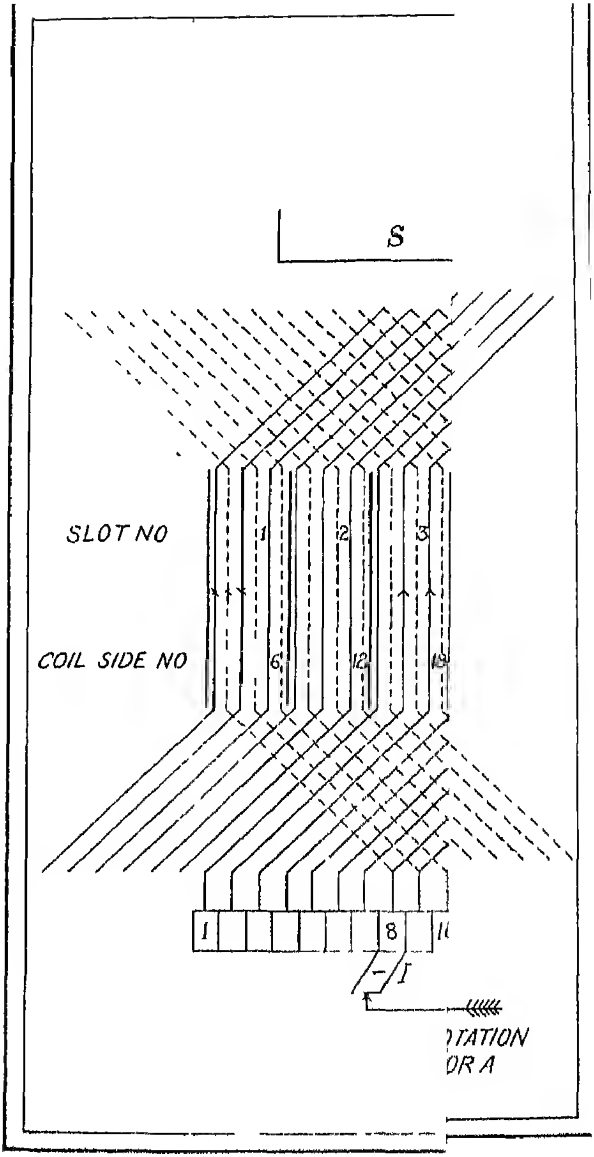
No. of coil-sides $= 51 \times 2 = 102$

Winding pitch (back and front)

$$= \frac{102 \pm 2}{4} = \frac{100}{4} = 25$$

Commutator pitch $= 25$

This gives a retrogressive winding.



S
SLOT NO
1
2
3
COIL SIDE NO
6
12
18
1
8
–
I

Winding Table

As the complete winding table is very big we give a part of this table and the student can complete it without much difficulty.

Back Connection	*Front Connection*
1 to 26	26 to 51
51 to 76	76 to 101
101 to 24	24 to 49
49 to 74	74 to 99
99 to 22	22 to 47
47 to 72	72 to 97
	etc. etc.

Draw the winding diagram as shown in fig. 76 and fix up the position of the brushes etc. as already explained in examples 3 and 4.

The distance between the two brushes here is $\frac{51}{4}=12\frac{3}{4}$ commutator bars.

CHAPTER XI

ARMATURE REACTION AND COMMUTATION

11-1. Armature Reaction.

It has been stated before that every current carrying conductor is surrounded by magnetic lines of force. Armature reaction is due to the magnetising effect of the currents in the armature conductors.

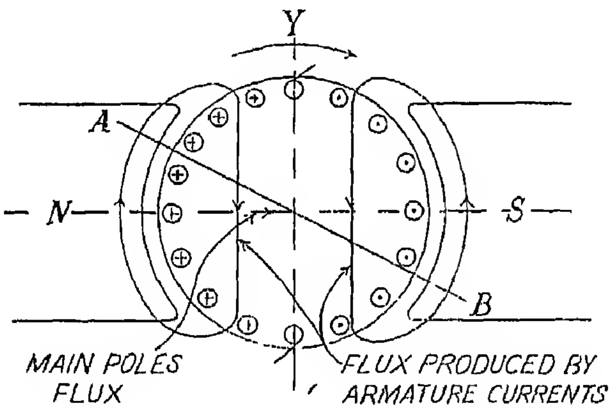


Fig. 77(*a*). YY′ is the geometrical neutral axis.

If the brushes are placed on the geometrical neutral axis (YY′), the armature currents magnetise the armature core along the line joining the brushes and therefore at right angles to the magnetisation produced in it by the main pole. In this figure the main field flux acts in the direction N to S. The flux produced by armature currents acts in the direction YY′ as shown. The resultant flux in the core is in the direction AB.

The resultant magnetisation therefore is oblique and it becomes more inclined with more armature current flowing in the conductors. This effect of armature currents is called the cross-magnetising effect.

Since the brushes must approximately be on the diameter at right angles to the lines of force through the armature (*i.e.*, on the magnetic neutral axis) it is therefore found necessary to shift the brushes forward (*i.e.*, give them lead) to obtain sparkless commutation.

The angle through which the brushes are moved is called the angle of lead. The direction of armature currents with the brushes moved forward becomes as shown in Fig. 77(*b*).

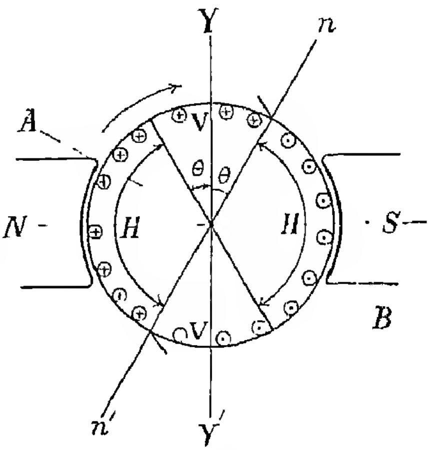


Fig. 77(*b*)
YY′ is Geometrical neutral axis.
nn′ is Magnetic neutral axis.
AB is the resultant direction of the lines of forces.

The armature field is no longer at right angles to the main field and the easiest way to consider its effect is to assume that it is the resultant of two components—one in the direction YY′ already considered and called the cross-magnetising component and the other in the direction SN directly opposing the main field poles and therefore called demagnetising effect. It is convenient in this case to conceive the whole winding divided into two groups—a vertically wound belt consisting of conductors in the belt VV consisting of top conductors which subtend an angle 2θ (where θ=angle of lead of the brushes) joined to the corresponding bottom conductors producing the demagnetising component of the armature flux.

The other belt consists of conductors in belt HH consisting of conductors under N pole which subtend the angle $(180—2\theta)$ oined to the corresponding conductors on the right. The belt HH produces a cross-magnetising effect as already explained.

The overall effect of armature reaction is that the main field is weakened to a certain extent and the induced emf. is reduced.

To minimize the effect of armature reaction two methods are generally used :

(*a*) Interpoles are provided. These are small auxiliary poles placed on the yoke between the main poles, and wound

with comparatively few turns connected in series with the armature. Their polarity, in the case of generator is the same as of the next main pole in the direction of rotation. When the interpoles are provided the brushes are not given any shift and are placed on the geometrical neutral axis of the machine.

(*b*) Compensating windings are used in the pole faces. These like interpoles are connected in series with the armature. They are arranged so that they carry currents in the direction opposite to the direction of current in the adjacent armature conductors.

11-2. Calculation of the cross magnetising and demagnetising amp-turns of the armature.

In the 2 pole machine shown in the figures, let :

Z = Total number of armature conductors

I_c = Current flowing in each conductor

θ = Angle of lead of the brushes in space degrees.

Note.—In a 2 pole machine space angle and electrical angle are equal.

Total armature amp. turns $= \dfrac{ZI_c}{2}$

Armature amp-turns per pole

$$= \frac{ZI_c}{2p}$$

where p = Number of poles.

Conductors in the belt VV (Fig. 77*b*)

$$= Z \times \frac{4\theta}{360} = Z \times \frac{2\theta}{180}$$

Demagnetising amp. turns per pole

$$= \frac{ZI_c}{2p} \times \frac{2\theta}{180} \qquad \text{...Eq. (11-1)}$$

Cross magnetising amp. turns per pole

$$= \frac{ZI_c}{2p} - \frac{ZI_c}{2p} \times \frac{2\theta}{180}$$

$$= \frac{ZI_c}{2p}\left[1 - \frac{2\theta}{180}\right] \qquad \text{...Eq. (11-2)}$$

In calculating the cross magnetising or demagnetising amp-turns per pole the student must be very careful about electrical and space angles. If the angle of lead is taken in space degrees then the pole pitch angle must be taken in space degrees. In a 4 pole machine the pole pitch is 90 space degrees and in a 10 pole machine the pole pitch is 36 space degrees. This is illustrated in the examples that follow.

Example 1. A 4 pole generator has a simple wave wound armature with 35 slots and 12 conductors per slot and delivers 40 amps on full load. If the brush lead is 9 space degrees find the demagnetising and cross magnetising amp-turns per pole at full load.

Total number of armature conductors

$$=35\times12$$
$$=420$$

Total armature amp. turns per pole

$$=\frac{ZI_c}{2p}$$
$$=420\times\frac{40}{2}\times\frac{1}{8}=1050$$

Note. $I_c=\frac{40}{2}$, as in a wave wound armature there are two paths for the flow of current.

Space angle per pole $=\frac{360}{4}=90°$

Demagnetising amp. turns per pole

$$=1050\times\frac{2\times9}{90}$$
$$=210\ \textit{Ans.}$$

Cross-magnetising amp. turns per pole

$$=1050-210=840\quad \textit{Ans.}$$

or Cross magnetising amp. turns per pole

$$=1050\left(1-\frac{2\theta}{90}\right)=1050\times\frac{72}{90}$$
$$=840.$$

If the angles are taken in electrical degrees then the brush shift of 9 space degrees in this case=18 electrical degrees.

Demagnetising amp. turns per pole

$$=1050\times\frac{2\times18}{180} \qquad \left[\begin{array}{l}180^\circ \text{ electrical}\\ =\text{pole pitch.}\end{array}\right]$$

$$=210.$$

Example 2. An 8 pole lap wound generator has 1,152 conductors on the armature and delivers 800 amps. at full load. The brushes have a lead so that the coil is short-circuited when it has advanced 15% of the pole pitch from the geometrical neutral axis. Find the demagnetising and cross magnetising amp. turns per pole at full load.

Armature amp. turns per pole

$$=\frac{ZI_c}{2p}$$

$$I_c=\frac{800}{8}=100 \text{ amps.}$$

Armature amp. turns per pole

$$=\frac{1152\times100}{16}=7200$$

Taking angles in electrical degrees

Pole pitch $=180^\circ$

Angle of lead $=180\times\frac{15}{100}=27^\circ$

Demagnetising amp. turns per pole

$$=7200\times\frac{2\times27}{180}$$

$$=2160$$

Taking angles in space degrees

Pole pitch $=\frac{360}{8}=45^\circ$

Angle of lead $=45\times\frac{15}{100}=6.75^\circ$

Demagnetising amp-turns per pole

$$=7200\times\frac{6{\cdot}75\times2}{45}$$
$$=2160$$

Cross-magnetising amp-turns per pole

$$=7200-2160=5040 \quad \textit{Ans.}$$

11-3. Commutation.

Commutation is that process whereby current in an armature coil as it passes from one side of the *axis of commutation* (Neutral zone axis) to the other, is reduced to zero, and a reverse current is built up equal in value to the current in the circuit of which the coil is about to become a part. This process should take place without sparking between the brushes and the commutator.

Let I amperes of current be flowing in a coil in clockwise direction when the short-circuit is about to start. If I amperes of current flows in the anticlockwise direction after the short-circuit is over, the commutation is said to be ideal. This ideal commutation is not realised in actual practice. Due to the self inductance of the short-circuited coil an emf. called the reactance voltage is induced which opposes the change of current in the short-circuited coil. The commutation then is said to be bad and this results in

1. Local heating of the brush.
2. Sparking between the brushes and the commutator.

11-4. Aids to commutation.

(*a*) The rise of current in an inductive circuit is given by

$$i=\mathrm{I}\left(1-e^{-\frac{\mathrm{R}t}{\mathrm{L}}}\right)=\mathrm{I}-\mathrm{I}e^{-\frac{\mathrm{R}t}{\mathrm{L}}}$$

$$-\frac{\mathrm{R}t}{\mathrm{L}}$$

In this expression I_e is the amount by which the current falls short of what it would be if L were zero. This amount can be decreased by (*i*) increasing R, (*ii*) increasing *t* or (*iii*) decreasing L.

(*i*) R is increased by using carbon brushes instead of copper. Carbon has a contact resistance about 12 times that

of copper. The drop per contact with carbon brushes lies between 1 and 2 volts for normal current densities.

(*ii*) t can be increased by using wider brushes enough to cover 3 or 2 segments.

(*iii*) L for the armature is constant and can be changed only by redesigning the winding.

(*b*) **Use of Interpoles or Commutating poles.** The best method of overcoming cross-magnetisation and commutation difficulties is by the use of interpoles. The field produced by the interpoles is opposite in direction to the armature field. These two fields can be made equal and they would remain nearly equal at all loads because both are produced by same current. The magnetic and geometrical neutral axes are made to coincide at all loads.

The required commutating emf. is obtained by putting extra turns on the interpoles. The extra field induces in the coil undergoing commutation an emf. which neutralizes the reactance voltage. The action is entirely automatic. Interpoles do not correct armature reaction but they do eliminate the demagnetising effect because the brush lead is not required.

Average value of Reactance voltage

$$= L \times \frac{2I}{T_c} \qquad \text{...Eq.(11-3)}$$

where T_c = Period of commutation

L = Inductance of the coil

I = Current in the coil before short-circuit and after short-circuit.

11-5. To find T_c *i.e.*, Time of short-circuit.

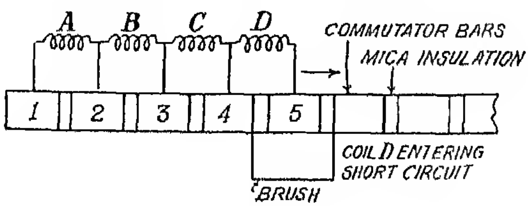


Fig. 78(*a*)

Let us consider a slot with 3 coils per slot, consider the 3 coils B, C, D shown in Fig. 77(*b*) the sides of which form a slot

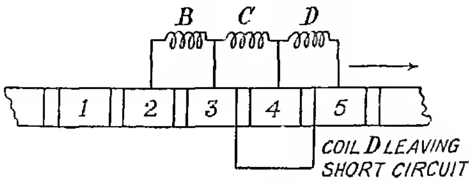


Fig. 77(*b*)

layer. The time of short-circuit is regarded as the period which elapses between the begining of short-circuit of coil D and the end of short circuit of coil B.

It will be noted from diagrams (*a*) and (*b*) that the distance moved by the commutator during the short-circuit of coil D.

$$=W_b-W_m.$$

where W_b=Brush width

W_m=Width of mica insulation

Diagram (*c*) illustrates the end of short circuit of coil B. The total distance moved by the commutator during the short

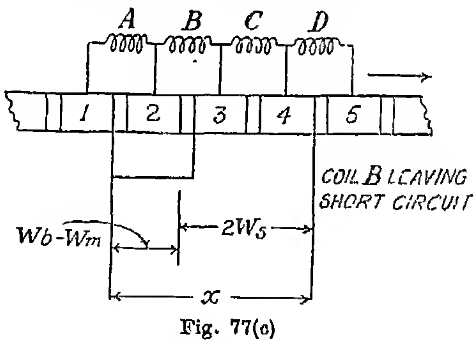


Fig. 77(*c*)

circuit of coils B, C and D is :

$W_b-W_m+2(W_s)$ or $W_b-W_m+(3-1)W_s$

where W_s=Pitch of commutator segment at the commutator surface

The general expression for the distance moved for N coils

$$=W_b-W_m+(N-1)W_s$$

Thus the time of short-circuit of a slot layer

$$T_c=\frac{W_b-W_m+(N-1)W_s}{V_c} \qquad \text{...Eq.(11-4)}$$

where V_c=Peripheral velocity of the commutator.

Example 3. An armature with 2 coils per slot rotates at 990 rpm. Number of commutator segments is 120, brush width $1\frac{1}{2}$ commutator segment pitch and thickness of mica insulation between segments is $\frac{1}{8}$ of the segment pitch. Find the time of short circuit of a slot layer.

Time of one revolution of commutator$=\frac{1}{900}$ min.

$$=\frac{60}{900}\text{ sec.}=\frac{1}{15}\text{ sec.}$$

Distance travelled in $\frac{1}{15}$ sec. by a point on the periphery of the commutator $=120W_s$.

Distance travelled in one sec.

$$=15\times120$$
$$=1800\ W_s.$$

$$T_c=\frac{W_b-W_m+(N-1)W_s}{V_c}$$

$$=\frac{1\frac{1}{2}W_s-\frac{1}{8}W_s+W_s}{1800\ W_s}$$

$$=\frac{2\frac{3}{8}\ W_s}{1800\ W_s}=\frac{19}{8}\times\frac{1}{1800}$$

$$=\frac{19}{14400}\text{ sec. } \textit{Ans.}$$

Example 4. In a D.C. generator with 60 slots and 2 coils per slot the commutator surface has a circumference of 60″.

Thickness of mica between segments is ·03 in. Width of the brush is ·75 in. Determine the period of short circuit. The machine has a speed of 875 r.p.m.

Peripheral speed of commutator

$$V_c=\frac{875}{60}\times 60=875 \text{ inches per sec.}$$

$$T_c=\frac{W_b-W_m+(N-1)W_s}{V_c}$$

Number of commutator segments $60\times 2=120$

$$W_s=\frac{60}{120}=\frac{1''}{2}$$

$$W_m=\cdot 03$$

$$W_b=\cdot 75$$

$$T_c=\frac{\cdot 75-\cdot 03+(2-1)\times\frac{1}{2}}{875}=\frac{1\cdot 22}{875} \text{ sec.}$$

Ans.

Example 5. The period of short-circuit of a coil of inductance 6 microhenry is $\frac{1}{1500}$ sec. The current per conductor is 100 amps. Determine the reactance voltage induced in the coil on the reversal of current.

The current changes from 100 amps. flowing in one direction to 100 amps. flowing in the opposite direction. Hence the total change of current is $2\times 100=200$ amps.

$$T_c=\frac{1}{1500} \text{ sec.}$$

$$E=\frac{6}{10^6}\times\frac{200}{\frac{1}{1500}}=\frac{6}{10^6}=200\times 1500$$

$$=1\cdot 8 \text{ volts.} \quad \textit{Ans.}$$

CHAPTER XII

CHARACTERISTIC CURVES OF D.C. GENERATORS

12-1. The behaviour of various types of dynamos can be best studied with the help of their characteristic curves. These, for generators, are curves connecting voltage and current when the machine is driven at a constant speed and are :

(*a*) **Open circuit characteristic** (OCC , sometimes called No load characteristic or Magnetisation curve, gives the relation between the emf. generated by the armature and field current under no load conditions.

(*b*) **External characteristic** gives the relation between the terminal *p. d.* across the machine and the current in the external circuit.

(*c*) **Total characteristic** gives the relation between the generated emf. and the armature current.

The first two curves can be obtained experimentally and the third one by graphical construction.

12-2. Open circuit characteristic and the shunt field resistance line.

Fig. 78 shows the O.C.C. of a shunt dynamo obtained by running it as a separately excited machine at a constant speed say N rpm.

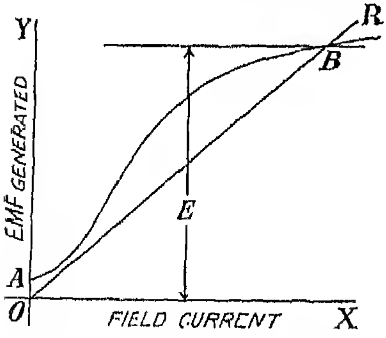


Fig. 78. O.C.C. of a shunt generator.

OA represents the emf. produced by the residual magnetism of the machine.

Suppose the shunt field resistance is x ohms. If a current I amps. is to pass through the shunt field the voltage needed is Ix volts. So plot a point I amp. along OX and Ix volts along OY. Join it with the origin O and produce it. The line so obtained is the field resistance line as shown by OR in the figure.

Suppose this line OR cuts the OCC at point B. This point

then gives the emf. (E) which the machine will generate at No load as a shunt generator when driven at N rpm.

12-3. Given the OCC at speed N, to obtain the OCC at speed N′.

If flux is constant we have :—

$$\frac{\text{emf. at speed N}'}{\text{emf. at speed N}}=\frac{E'}{E}=\frac{N'}{N}$$

Draw an ordinate RQP.

RP=emf. at speed N
=E

QP=emf. at speed N′
=E′

$$\frac{QP}{RP}=\frac{E'}{E}=\frac{N'}{N}$$

$$QP=RP\times\frac{N'}{N}$$

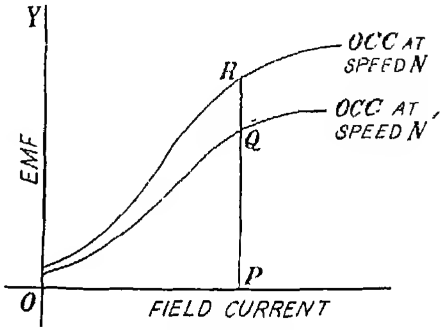


Fig. 79

A number of similar other points of the OCC for speed N′ can be calculated in the same way and the OCC for this speed drawn.

12-4. Critical resistance of the shunt field circuit.

Figure 80 shows the OCC at a constant speed N and the shunt field resistance line OR cutting the OCC at point B.

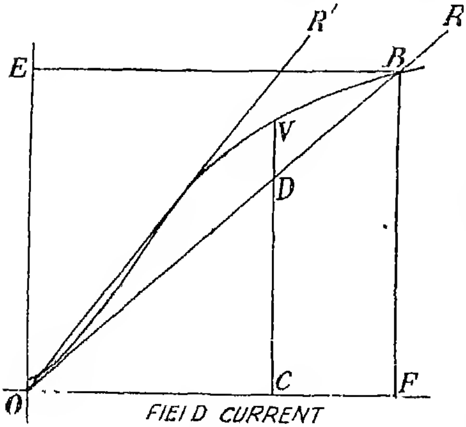


Fig. 80

The emf. that the machine will generate is represented by point B=OE.

Let us consider any other point say C on the base line OX. The shunt field current is OC. The voltage needed to send this current through the shunt field is CD. The emf. generated by the machine is CV which is greater than CD. Therefore the shunt field current rises and excites the

field further. Only at point B the emf. generated equals the field current OF multiplied by the field resistance and here the conditions are stable.

Effect of changing the field resistance.

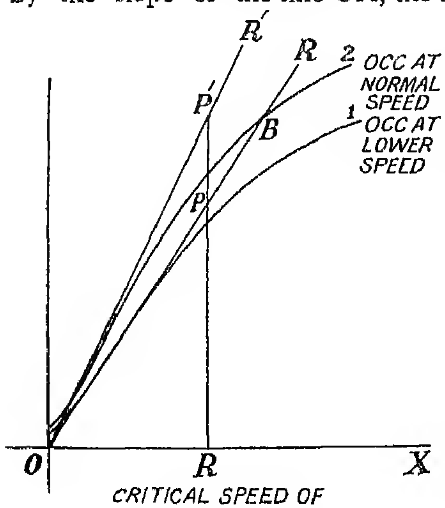
(*a*) Suppose the regulator resistance is decreased. The slope of the line OR decreases and the point B moves up forward on the curve. The emf. OE increases.

(*b*) Suppose the regulator resistance is increased. The slope of the line OR increases and point B moves down the curve towards the origin and the voltage OE decreases.

(*c*) *Critical Resistance.* Let the resistance of the field be increased till the slope of the line has increased to such an extent as to become tangent to the curve. It now takes the position OR′. The slope of OR′ gives the critical value of the field resistance at the given speed N because if the resistance is increased beyond this value, the machine will fail to excite.

12-5. Critical speed of a shunt generator.

If the shunt field resistance of a generator is represented by the slope of the line OR, the machine if started from rest will begin to excite at a speed such that the line OR is tangent to the OCC at that speed. This OCC is shown by the curve O1. When the speed has risen to the normal speed the OCC takes the shape as shown by the cruve O2 which the line OR cuts at the point B.



CRITICAL SPEED OF A SHUNT DYNAMO
Fig. 81

At speeds lower than those represented by the curve O1 the machine will not build up voltage. The speed represented by the curve O1 is known as the critical speed.

To obtain the critical speed.

Draw line OR′ representing the critical resistance at the normal speed. Draw an ordinate cutting line OR′ at P′, OR at P and OX at R. Then if N is the normal speed and N_c the critical speed

$$\frac{N}{N_c}=\frac{P'R}{PR}$$

$$N_c=\frac{N\times PR}{P'R}.$$

12-6. External characteristic of a separately excited dynamo.

OE is the emf. (E) generated by the machine at some constant speed N and is the terminal *p. d.* at no load. If the

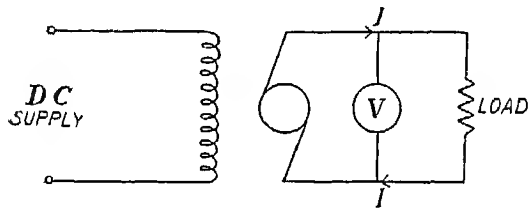


Fig. 82

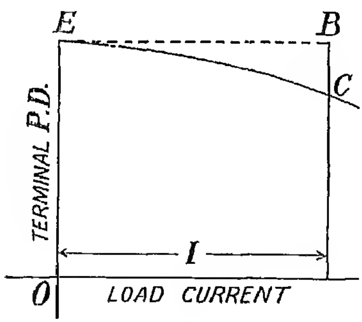


Fig. 83

machine is loaded the terminal *p. d.* would fall due to

(*a*) Armature resistance drop.

(*b*) The weakening of the field due to armature reaction.

The terminal *p. d.* falls as shown by the curve EC and at a load current I amps. the fall of voltage equals BC.

If the armature reaction drop is neglected the terminal *p. d.* would be $E-I_a R_a$ where R_a is the armature resistance and I_a the armature current (and also the load current in this case).

12-7. External characteristic of a shunt dynamo.

The drop of voltage with load in this case is more rapid than for a similar, but separately excited machine. This is

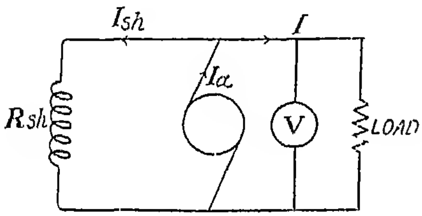


Fig. 84

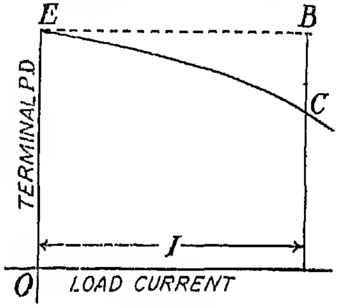


Fig. 85

because in addition to the two causes for drop in terminal *p. d.* the field current of the shunt machine decreases as the *p. d.* falls, thus further weakening the field.

Here $I_a = I + I_{sh}$

where I_{sh} = Field current.

12-8. External characteristic of a series dynamo.

AB is the OCC of the series dynamo when excited sepa-

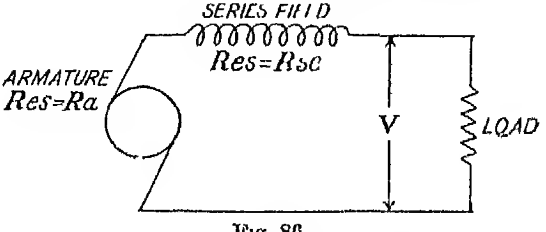


Fig. 86

rately. In the series dynamo the load current is also the exciting current and hence the OCC would have been the external

characteristic also if there had been no voltage drops as stated below :

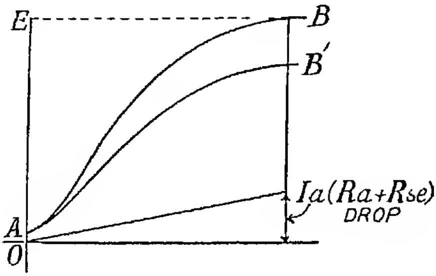


Fig. 87

(*a*) Armature and series field resistance drop.

(*b*) Armature reaction drop.

If armature reaction drop is neglected then the terminal p. d. at any load current $=E-Ia(Ra+Rse)$

$=$OE(no load voltage or emf.)

$-Ia(Ra+Rse)$

where $Rse=$Res. of series field.

The $Ia(Ra+Rse)$ line is shown in the diagram and if the drops at various values of the current are subtracted from the OCC the external characteristic AB′ is obtained (neglecting armature reaction). Thus we see that in the case of a series dynamo the terminal *p. d.* rises as the load increases.

12-9. External characteristic of compound generator.

(*a*) Short-shunt compound dynamo.

$$Ia=I+Ish$$

$$Ish=\frac{V+IRse}{Rsh}$$

$$E=V+IRse+IaRa$$

(neglecting armature reaction.)

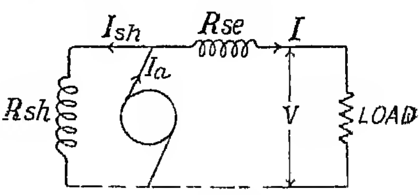


Fig. 88

(*b*) Long shunt compound dynamo.

$$Ish=\frac{V}{Rsh} \quad \text{and} \quad Ia=I+Ish$$

$E=V+Ia(Ra+Rse)$ neglecting armature reaction.

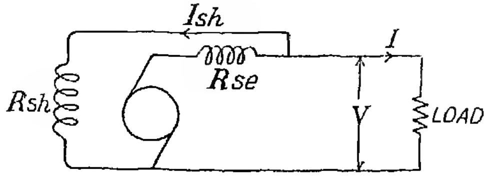


Fig. 89

In a compound generator if the terminal voltage at full load is the same as at no load as shown by curve 1 the machine is said to be "Level compounded." This means that the effect of series field has been to cause a rise in terminal *p. d.* at full load by the same amount as the fall in terminal *p. d.* by the various causes.

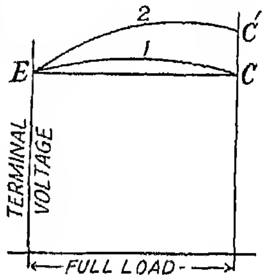


Fig. 90

Curve 2 is the external characteristic of an "over compounded" generator. Rise in voltage due to the series field is more than the voltage drop due to the various causes.

12·10. Application of different types of generators.

Series generators are used for lighting arc lamps, single or a number in series. It is also used as a voltage booster in certain types of distribution systems.

Separately excited generators are used for special purposes only such as electroplating.

Shunt generators with field regulators are universally used for all ordinary lighting and power purposes. It is also used for charging batteries. This is practically a constant voltage machine, the slight drop in voltage being adjustable by the field regulator.

Compound generators over-compounded are used where the load lies at the end of a transmission line and specially for traction purposes for compensating line drops with changed loads.

Example 1. The magnetisation curve for a separately excited generator running at normal speed and excited from a 230V supply is :—

Field current	0	·2	·4	·6	·8	1	1·2	1·4	1·6
Induced emf.	4	66	128	179	213	240	257	271	279

Field current	1·8	2
Induced emf.	285	291

Determine the field circuit resistance necessary to give a no load voltage of 275 What no load voltage would be obtained

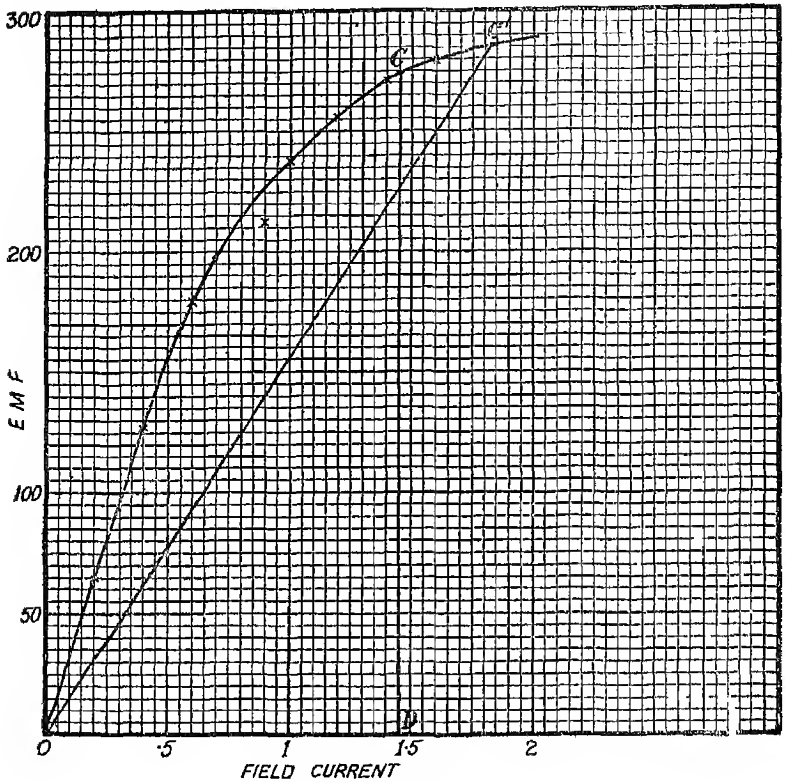


Fig 91

with the same value of field circuit resistance with the machine shunt connected.

1. Plot the open circuit characteristic.

2. To find the field circuit resistance to give a no load emf. 275V. When separately excited draw CD from point C on the curve reading 275V to cut OX at D. The field current needed for this emf. is 1 45 amps. The excitation voltage being 230V, the field circuit resistance$= \frac{230}{1{\cdot}4} = 156$ ohms.

3. To find the emf. induced at no load if the machine is shunt connected with the field circuit resistance of 156 ohms draw the 156 ohm. resistance line OC′ to cut the OCC at the point C′. Point C′ reads 286V.

Open circuit voltage as shunt generator=286V. *Ans.*

Example. 2. The O.C.C. of a separately excited dynamo driven at 500 rpm. is as follows :—

Field Current	1	1·5	2	2·5	3	3·5	4	4·5
Emf. Volts.	110	155	186	212	230	246	260	271

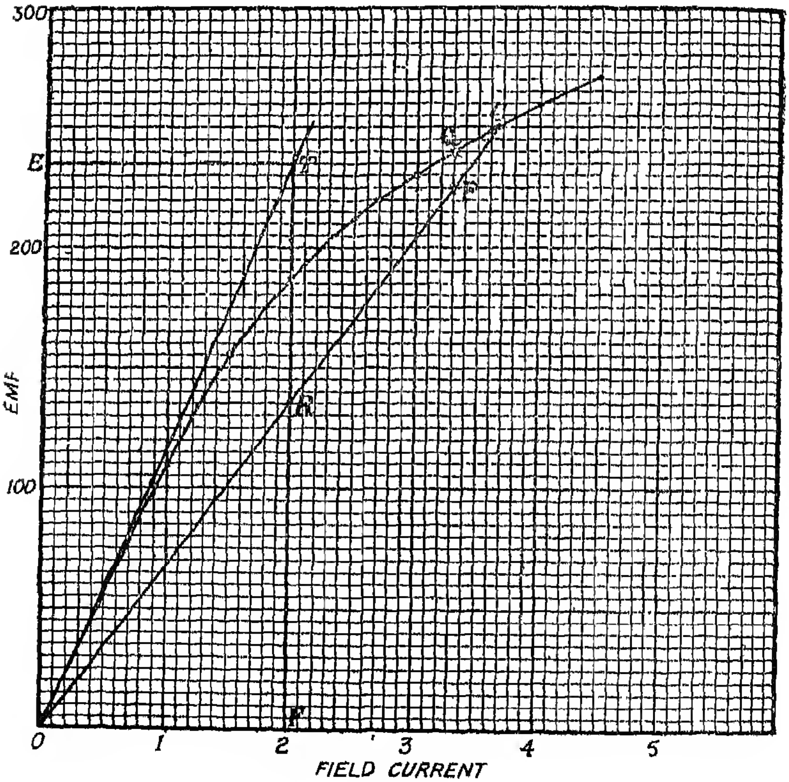


Fig. 92

The machine is connected as a shunt dynamo and driven at 500 rpm. Find :—

(*a*) Emf. to which the machine will excite when the field circuit resistance is 68 ohms.

(*b*) The additional resistance required in the field circuit to reduce the emf. to 220V.

(*c*) Critical value of the shunt field resistance.

(*d*) Critical speed when the field circuit resistance is 68 ohms.

(*e*) The speed at which an emf. of 225V can be obtained with field circuit resistance 68 ohms. (Neglect voltage drop in the armature due to shunt field current).

Plot the open circuit curve.

(*a*) Draw the 68 ohm. resistance line OC. It cuts the OCC at the point C which reads 250V.

(*b*) At 220 volts the OCC reads 2·7 amps. as the field current.

Res. of field circuit to give 220V emf. $=\frac{220}{2\cdot7}=81\cdot4$ ohms.

Additional res. required$=81\cdot4-68=13\cdot4$ ohms.

(*c*) Draw OT tangential to the OCC.

Critical resistance of the field circuit

$$=\frac{TF}{OF}=\frac{236}{2}=118 \text{ ohms.}$$

(*d*) The vertical line TF cuts the critical resistance line and the 68 ohm. resistance line at T and R respectively. Then

$$\frac{\text{Critical speed } N_c}{\text{Speed}}=\frac{RF}{TF}=\frac{136}{236}$$

or

$$\frac{N_c}{500}=\frac{136}{236}$$

$$N_c=500\times\frac{136}{236}=288 \text{ rpm.}$$

(*e*) It is required to reduce the open circuit voltage by speed regulation, to 225V. Through 225 volt point on the vertical axis draw a line to cut line OC at P. The OCC

at new speed must pass through P. Draw a vertical line through P to cut the OCC at Q. Read off the voltage at Q. It is 240 volts.

$$\text{Then required speed} = \frac{225}{240} \times 500$$

$$= 470 \text{ r.p.m.} \quad \textit{Ans.}$$

Example 3. The external characteristic of a shunt generator is :—

Load Current amps.	0	5	15	20	25	30
Terminal volts.	100	98	93	88	82	73

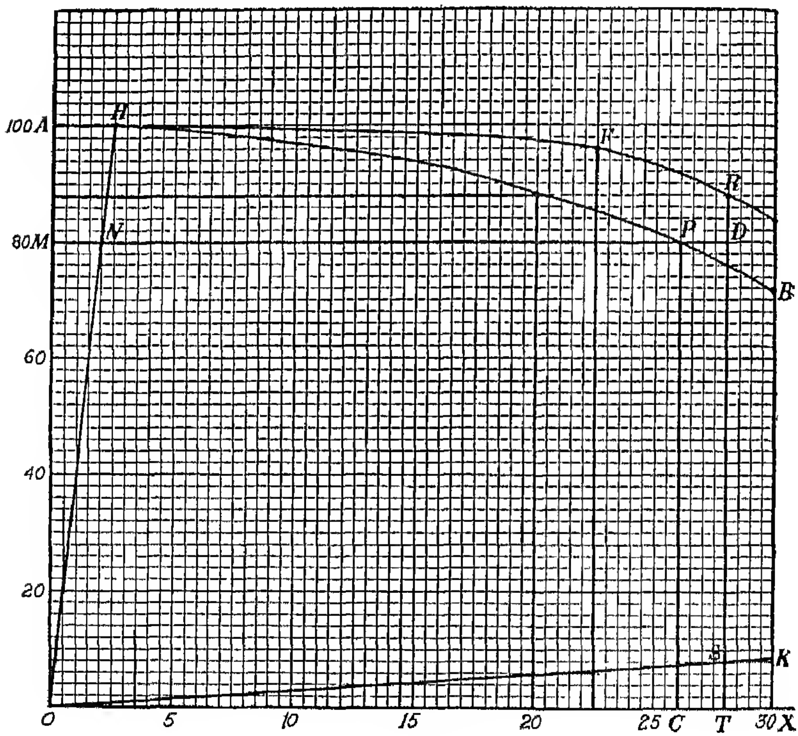


Fig 93

If armature resistance is ·3 ohm. and shunt field resistance 40 ohms. draw the total characteristic.

Plot the external characteristic APB as shown.

At 30 amp. armature current the armature drop is $30 \times \cdot 3 = 9$ V. At 30 amp. mark the point K representing 9 volts. Join OK. This gives the armature drop line OK. At say 80 volts the field current is 80/40 or 2 amps. Mark MN=2 amps. Join ON and produce. This gives the field resistance line OH.

Take a point P on the external characteristic and draw PC $\perp$ to OX and PNM parallel to OX. MN is the shunt field current at the voltage represented by point P. Take CT=MN.

Therefore armature current

$$I_a = OC + CT = OT$$

Voltage drop in armature at current OT

$$= ST$$

Emf. generated when armature current is OT

$$= PC + ST = DT + RD = RT$$

Point R therefore lies on the total characteristic curve.

Plot a few more points in a similar manner and draw the curve AFR.

Example 4. A shunt generator has the following O.C.C. at 600 r.p.m.

Field current	·5	1	1·5	2	2 5
Emf. generated (volts.)	100	172	209	235	253

The armature resistance is ·2 ohm and the shunt field resistance 110 ohms. Draw the external characteristic for the machine when run at 600 r.p.m.

1. Plot the O C.C. from the points given.

2. Draw the 110 ohms shunt field resistance line cutting the O.C.C. at point A. Then point A gives the no load voltage of the machine.

3. Take any point P on the OCC. and draw the perpendicular PR cutting the shunt field resistance line at Q.

At the instant P the terminal voltage is represented by

QR=220 volts.

Emf=PR=235 volts.

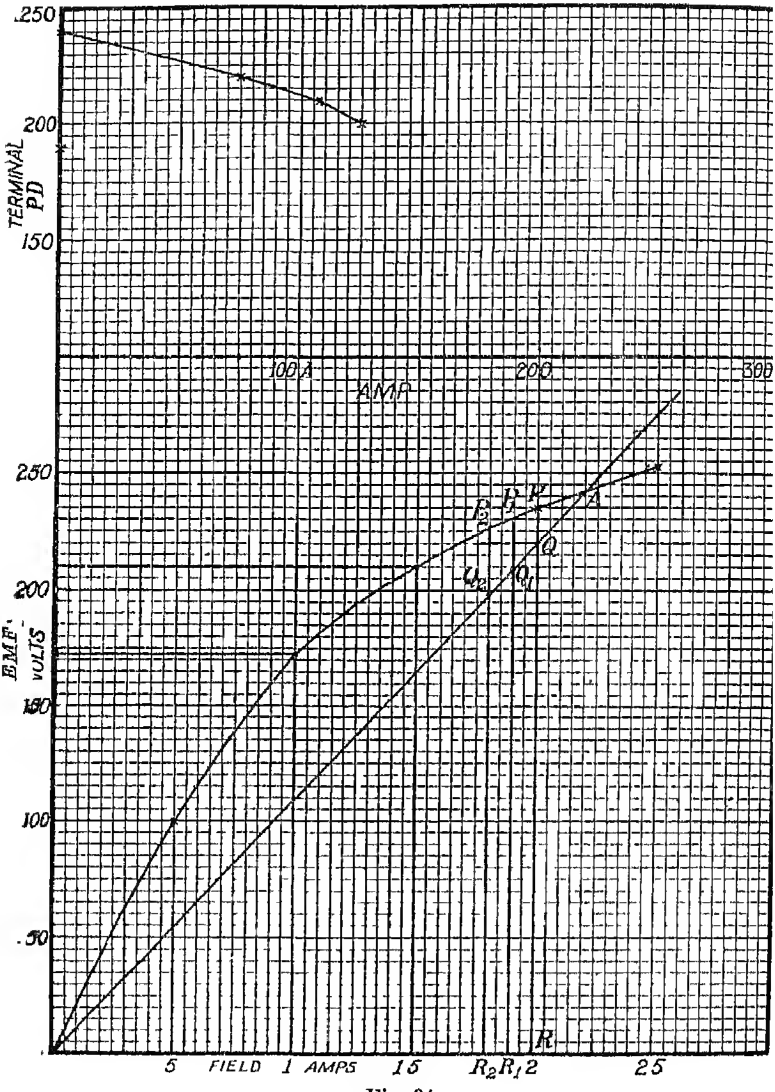

250
200
TERMINAL PD
150
100 A
200
300
AMP
250
200
E.M.F. VOLTS
150
100
50
P_2 P_1 P
A
Q
Q_2
Q_1
R
5
FIELD
1
AMPS
15
$R_2 R_1$ 2
25

Fig. 94

Neglecting armature reaction PQ represents the drop in the armature $=I_aR_a$

$$PQ=15 \text{ volts.}$$

$$R_a=\cdot 2 \text{ ohm.}$$

$$I_a=\frac{15}{\cdot 2}=75 \text{ amps.}$$

Load current $=75-I_{sh}=75-2=73$A.

This gives a point on the external characteristic representing 73A and 220 volts.

Similarly point P_1 can be taken.

$$Q_1R_1=210 \text{ V.}$$

$$P_1R_1=232 \text{ V.}$$

$$P_1Q_1=22 \text{ V.}$$

$$I_a=\frac{22}{\cdot 2}=110 \text{ A.}$$

$$I_{sh}=1\cdot 9 \text{ A. say } 2 \text{ A.}$$

External load current$=110-2=108$ A.

This gives another point on the external characteristic 108 A, 210 V.

The 3rd point is P_2

$$Q_2R_2=200 \text{ V.}$$

$$P_2R_2=225 \text{ V.}$$

$$P_2Q_2=25 \text{ V.}$$

$$I_a=\frac{25}{\cdot 2}=125 \text{ A.}$$

$$I=125-2 \text{ (approx.)}=123 \text{ A.}$$

The 3rd point, therefore, is 123 A, 200 V.

Similarly other points on the external characteristic can be obtained.

The characteristic has been shown with the three points obtained above.

Example 5. The open circuit voltage of a 4 pole constant speed D.C. generator varied with the field current as shown below.

Field current (amps.)	1	2	3
O.C. Voltage.	180	225	235

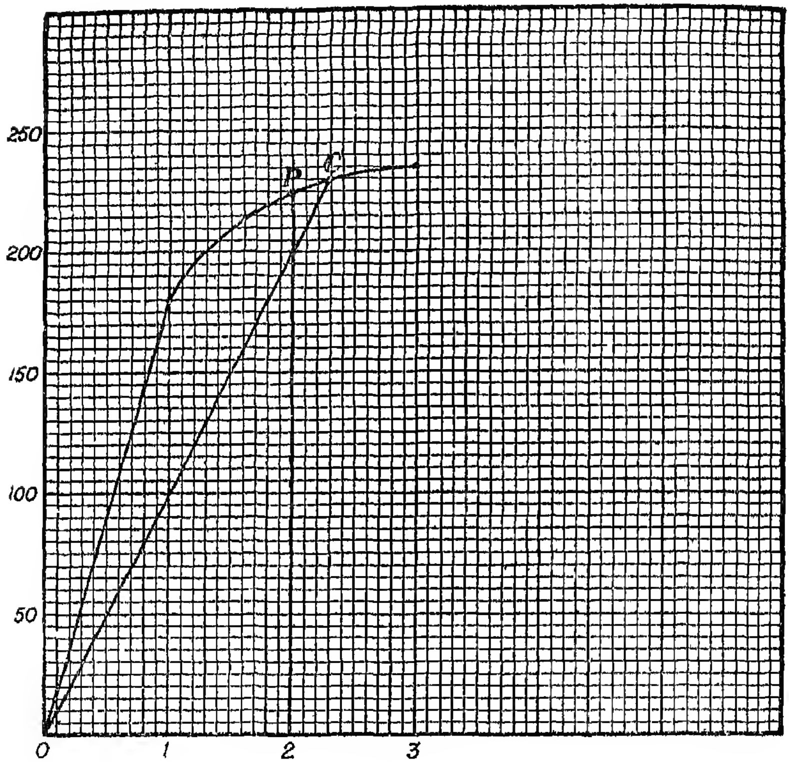


Fig. 95

(*a*) The machine is run as a shunt generator with the field resistance of 100 ohms. Find its open circuit voltage.

(*b*) If the armature resistance is ·15 ohm., calculate the load current for a terminal voltage of 200 volts. Neglect armature reaction.

(*c*) If with the terminals short-circuited, the armature current is 40 amps., calculate the flux per pole of residual magnetism. The armature is wave wound with 222 conductors and speed is 600 r.p.m.

(*a*) Draw the O.C.C. as shown and draw the 100 ohm shunt field resistance line taking a current of 2 amps. at 200 volts. The resistance line cuts the O.C.C. at point C representing 230 V.

(*b*) When terminal p.d. is 200 volts, the p.d. across the shunt field is also 200 volts. The emf. generated by the armature at this instant is 225 V. (as shown by point P on the O.C.C.).

∴ Armature drop (neglecting armature reaction)

$$=225-200=25 \text{ volts.}$$

$$I_a=\frac{25}{R_a}=\frac{25}{\cdot 15}=167 \text{ amps.}$$

The load current =armature current—field current

$$=167-2=165 \text{ amps.}$$

(*c*) On short-circuit the emf. circulating the current of 40 amps. in the armature $=40\times\cdot 15$

$$=6 \text{ volts.}$$

This is the emf. produced by the residual magnetism

$$\therefore \qquad 6=\frac{4\times\phi\times 222\times 600}{10^8\times 2\times 60}$$

or $\phi=0\cdot 135\times 10^6$ lines per pole.

Example 6. A D.C. generator gave the following open circuit characteristic :

Field Current	·25	·5	75	1	1·25	1·5	1·75
O.C. Voltage	54	107	152	185	210	230	245

The armature and field resistances are ·1 and 160 ohms respectively. Find :

(*a*) The voltage to which the machine will excite when run as a shunt machine at the same speed.

(*b*) The voltage lost due to armature reaction when 100 amps. are passing in the armature at a terminal p. d. of 175 volts.

(*c*) Percentage reduction in speed for the machine to fail to excite on open circuit.

Plot the O.C.C.

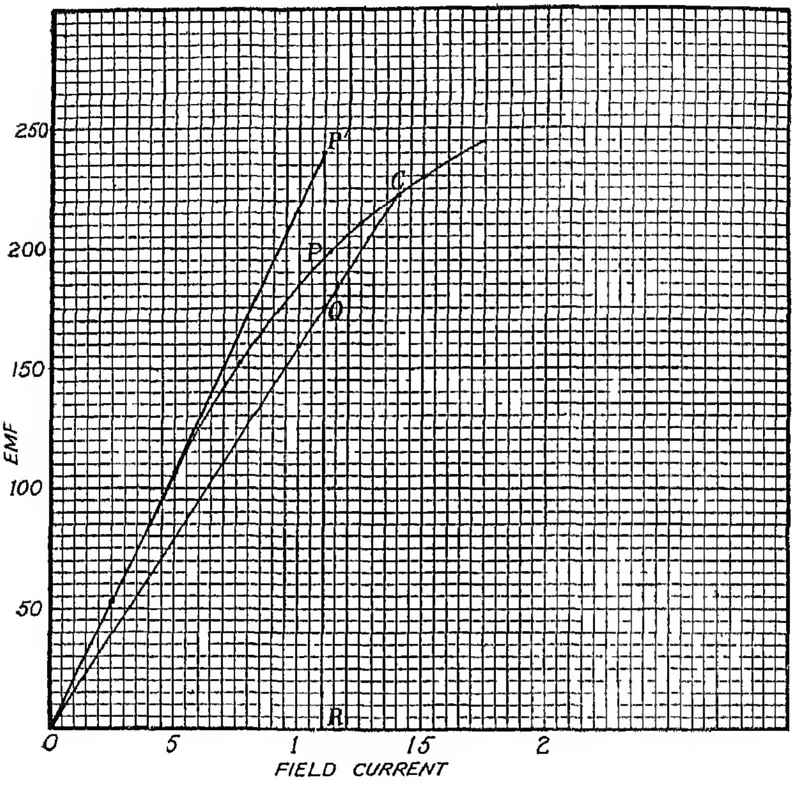


Fig. 96

(*a*) Draw the 160 ohm. field resistance line, plot a point 160 V, 1 A. Join with the origin and produce it to cut the O.C.C. at point C. This point reads 222 V. Therefore the machine will excite to 222 V. on open circuit as a shunt generator.

(*b*) When the voltage across the terminals is 175 V. the voltage across the field is also 175 V. Point Q represents this

point Draw the perp. line PQR. The emf. induced in the armature PR=197 volts.

$$PR-QR=197-175=22 \text{ volts.}$$

$$I_aR_a \text{ drop.}=100\times\cdot 1=10 \text{ volts.}$$

Voltage drop due to armature reaction

$$=22-10=12 \text{ volts.}$$

(*c*) Draw a line tangent to the O.C.C. as shown by OP′. From P′ draw perpendicular cutting the shunt field resistance line at Q. Then

$$\frac{N}{N_c}=\frac{\text{Speed}}{\text{Critical speed}}=\frac{P'R}{QR}=\frac{240}{175}$$

or

$$\frac{N_c}{N}=\frac{175}{240}=\frac{35}{48}$$

$$N_c=N-\frac{13}{48}N$$

Percentage reduction in speed

$$=100\times\frac{13}{48}=27. \quad \textit{Ans.}$$

CHAPTER XIII

LOSSES AND EFFICIENCY OF DYNAMOS

13-1. A dynamo is a machine for converting mechanical energy into electrical energy. When such a conversion takes place, certain losses occur (which are dissipated in the form of heat). These losses can be grouped as under :

(A) The copper losses (sometimes called electrical losses).

These are:—

(*a*) Armature copper loss $= I^2_a R_a$ watts.

(*b*) Field copper loss

In shunt field $= I^2_{sh} R_{sh}$ watts

In series field $= I^2_{se} R_{se}$,,

In interpoles $= I^2_a R_i$,,

(*c*) Brush contact resistance loss. This is usually taken into account by including brush contact resistance with the armature resistance.

(B) Iron and Friction losses (sometimes called the rotational losses). These are :—

(*a*) Iron losses

(*i*) Hysteresis loss in the armature $\propto NB_{max}^{1 \cdot 6}$.

(*ii*) Eddy current loss in the armature and pole shoes $\propto N^2 B_{max}^2$.

(*b*) Friction losses

(*i*) Brush friction

(*ii*) Bearing friction

(*iii*) Windage loss due to rotation of armature.

Some of the losses stated above remain practically constant at all loads and others are variable. This enables them to be grouped under the following two heads :

(A) Constant losses = Iron and Friction losses + shunt field losses

(B) Variable losses = copper losses − shunt field losses.

13-2. Efficiency.

The total mechanical input to the generator is not converted to electrical power but a part is lost in friction, windage and iron losses Further, the net electrical power output is not the same as the total electrical power generated. It is less than the electrical power generated by an amount spent in the copper losses. This conversion can be represented in the form of a diagram shown below :

A			B				C
Mechanical power Input	−	Iron and Friction losses	=	Electrical power developed in armature ($=EI_a$ watts)	−	Copper losses	=	Electrical power output (VI watts)

$$A = \text{B H.P. of driving engine} \times 746 \text{ watts}$$

$$B = \text{emf. of machine} \times I_a \text{ watts} = EI_a \text{ watts}$$

$$C = \text{Terminal p.d.} \times \text{Current output} = VI \text{ watts.}$$

Commercial or overall efficiency

$$= \frac{C}{A} = \frac{\text{Electrical output}}{\text{Mechanical Input}}$$

Electrical Efficiency $= \dfrac{C}{B} = \dfrac{\text{Electrical output}}{\text{Elec. power developed}}$

Mechanical Efficiency $= \dfrac{B}{A} = \dfrac{\text{Electrical power developed}}{\text{Mechanical Input}}$

The efficiencies are always expressed in percentage. Unless otherwise stated the efficiency is understood to mean the overall efficiency.

Overall Efficiency $= \dfrac{\text{Output}}{\text{Input}} = \dfrac{\text{Output}}{\text{Output}+\text{Losses}}$

$$= \frac{\text{Input}-\text{Losses}}{\text{Input}}$$

13-3. Maximum Efficiency.

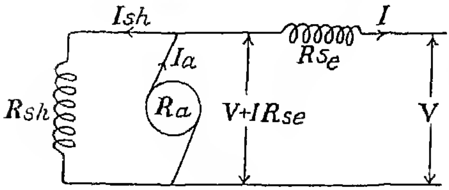


Fig. 97

Let us take the case of a short shunt compound machine, and determine the condition for maximum efficiency.

Variable losses $=I^2R_{se}+I_a^2R_a$

$=I^2R$ (approx.) where $R=R_{se}+R_a$

Constant losses P $=$ Iron and friction losses + shunt field loss.

$$\text{Efficiency } \eta = \frac{\text{Output}}{\text{Output}+\text{Losses}}$$

$$= \frac{VI}{VI+I^2R+P} \text{ (approx.)}$$

$$\frac{d\eta}{dI} = \frac{VI(V+2IR)-V(VI+I^2R+P)}{(VI+I^2R+P)^2}$$

Efficiency is maximum when

$$\frac{d\eta}{dI}=0$$

or $\qquad VI+2I^2R=VI+I^2R+P$

or $\qquad I^2R=P$

Efficiency is maximum when the variable and constant losses are equal. This condition holds good for any other type of machine.

Example 1. A shunt dynamo gives an output of 200 amps. at 500 volts. The armature resistance is 0·04 ohm and the resistance of the field winding is 100 ohms. What is the electrical efficiency of the machine. If it takes 150 H.P. to drive this generator, find its overall efficiency.

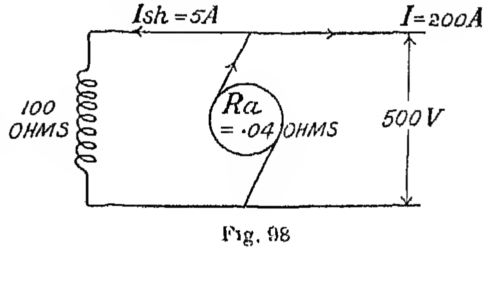


Fig. 98

$$I_{sh}=\frac{500}{100}=5 \text{ amps.}$$

$$I_a = 200 + 5 = 205 \text{ amps.}$$

Voltage drop in the armature

$$= 205 \times \cdot 04 = 8 \cdot 2 \text{ volts.}$$

Induced emf. in armature

$$E = 500 + 8 \cdot 2 = 508 \cdot 2 \text{ V.}$$

Electrical power developed

$$= EI_a$$
$$= 508 \cdot 2 \times 205 = 104181 \text{ watts.}$$

Power output $= 500 \times 200 \;\; = 100000$

Copper losses $= 104181 - 100000 = 4181$ watts.

Electrical efficiency $= \dfrac{100000}{104181} \times 100 = 96\%.$

Mechanical power input

$$= 150 \times 746 = 111900 \text{ watts.}$$

Iron, friction and windage losses

$$= 111900 - 104181 = 7719 \text{ watts.}$$

Overall efficiency $= \dfrac{100000}{111900} \times 100 = 89 \cdot 3\%.$

Example 2. A 500 volt long-shunt compound generator has the constant losses (shunt field and rotational) of 18 Kw. The resistances are—armature winding ·01 ohm, series field ·002 ohm, shunt winding 33 ohms. Find the maximum efficiency and the load at which it occurs.

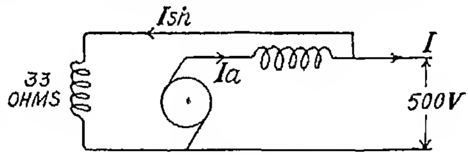


Fig. 99

Variable losses $= I^2_a \, (R_a + R_{se}) = I^2_a R$

where $R = R_a + R_{se}$

For maximum efficiency variable losses = constant losses

or $\quad I^2_a \, (\cdot 01 + \cdot 002) = 18 \times 1000$

or $\quad I^2_a = \dfrac{1800}{\cdot 012} = 1500 \times 1000$

$\quad I_a = \sqrt{150 \times 10^4} = \sqrt{150} \times 100$

$\quad = 1225$ amps.

External load current $I = 1225 - I_{sh}$

$$= 1225 - \frac{500}{33} = 1210 \text{ amps.}$$

External load at which the efficiency is a max.

$$= \frac{1210 \times 500}{1000} = 605 \text{ Kw.}$$

Max. efficiency $= \frac{\text{Output}}{\text{Output} + \text{Losses}} = \frac{605}{605 + 18 + 18}$

$$= \frac{605}{641} = 94{\cdot}4\%.$$

Example 3. A 3 Kw., 230 V. D.C. generator has the following particulars :—

Armature and brush contact resistance

$= 2{\cdot}2$ ohms.

Shunt field resistance $= 460$ ohms.

Series ,, ,, $= {\cdot}25$ ohms.

When the machine is run light as a motor at its normal speed and rated voltage it takes an armature current of ·6 amp. Find the electrical, mechanical and over-all efficiency of the machine when running as a long shunt generator at full load. The resistances given are at the normal working temperature.

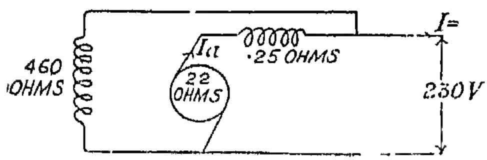


Fig. 100

Full load current I $= \frac{3 \times 1000}{230} = 13$ amps.

Shunt field current $= \frac{230}{460} = {\cdot}5$ amp.

Shunt field loss $= {\cdot}5 \times 230 = 115$ watts.

Armature current at full load

$$I_a = 13 + {\cdot}5 = 13{\cdot}5 \text{ A.}$$

Armature and series field loss

$$=13{\cdot}5^2\times(2{\cdot}2+{\cdot}25)$$
$$=13{\cdot}5^2\times2{\cdot}45=446{\cdot}5 \text{ watts.}$$

Iron and friction losses $=230\times{\cdot}6=138$ watts.

Electrical power output at full load

$$=3 \text{ Kw.}=3000 \text{ watts.}$$

Elec. power developed in armature at full load

$$=3000+\text{shunt field loss}$$
$$+\text{armature and series field loss}$$
$$=3000+115+446{\cdot}5=3561{\cdot}5 \text{ watts.}$$

Mechanical power input at full load

$$=3561{\cdot}5+\text{iron, friction losses}$$
$$=3561{\cdot}5+138=3699{\cdot}5 \text{ watts.}$$
$$=3700 \text{ watts. (approx.)}$$

$$\text{Electrical efficiency}=\frac{\text{Elec. output}\times100}{\text{Elec. power developed}}$$
$$=\frac{3000}{3561{\cdot}5}\times100=84{\cdot}2\%$$

$$\text{Mechanical efficiency}=\frac{\text{Elec. power developed}}{\text{Mechanical input}}\times100$$
$$=\frac{3561{\cdot}5}{3700}\times100=96{\cdot}2\%$$

$$\text{Overall efficiency}=\frac{\text{Electrical output}}{\text{Mechanical input}}\times100$$
$$=\frac{3000}{3700}\times100=81{\cdot}1\%. \quad \textit{Ans.}$$

Example 4. With the following particulars of a shunt machine find the rating and efficiency :

(*i*) As a generator

(*ii*) As a motor

(*a*) Supply voltage $=460$ V.

(*b*) Field current $=1$ A.

(*c*) Armature current $=51$ A.

(*d*) Armature resistance

$= \cdot 5$ ohm.

(*e*) Iron, friction and windage losses

$= 800$ watts.

(*f*) Emf. at 1000 rpm. $= 460$ V.

(*g*) Commutator voltage drop

$= 2$V

The currents under (*b*) and (*c*) are assumed to be the same for both the cases of generator and motor and losses under (*e*) are assumed constant.

(*i*) Generator armature resistance drop

$= 51 \times \cdot 5 = 25{\cdot}5$ V.

Commutator voltage drop

$= 2$ V.

$\therefore$ Emf of generator $= 460 + 25{\cdot}5 + 2 = 487{\cdot}5$ V

Speed $= \dfrac{487{\cdot}5}{460} \times 1000 = 1060$ rpm.

$$\left[\frac{E_2}{E_1} = \frac{N_2}{N_1}\right]$$

Losses

Armature copper loss $= 51^2 \times \cdot 5 = 1300{\cdot}5$ watts.

Commutator contact loss

$= 51 \times 2 = 102$ watts.

Field loss $= 1 \times 460 = 460$ watts.

Iron, friction and windage losses

$= 800$ watts.

Total losses $= 2662{\cdot}5$ watts.

Output current $=$ armature current $-$ field current

$= 51 - 1 = 50$ A. at 460 V.

Output $= \dfrac{VI}{1000} = \dfrac{460 \times 50}{1000} = 23$ Kw.

Input $= 23 + 2{\cdot}6625$ Kw. $= 25{\cdot}6625$

Efficiency $=\dfrac{\text{Output}}{\text{Input}}$

$=\dfrac{23}{25{\cdot}6625}\times100=89\ 6\%$

Rating 23 Kw, 460 V, 50 A, 1060 rpm. as generator.

(*ii*) Motor

Armature resistance drop same as for generator

$=51\times{\cdot}5=25{\cdot}5$ V

Commutator voltage drop

$=2$ V

Back emf. of motor $=460-27{\cdot}5=432{\cdot}5$ V

Speed as motor $=\dfrac{432{\cdot}5}{460}\times1000=940$ rpm.

Losses

Armature copper loss $=51^2\times{\cdot}5=1300{\cdot}5$ watts.

Commutator contact loss

$=51\times2=102$ watts.

Field loss $=1\times460=460$ watts.

Iron, friction and windage loss

$=800$ watts.

Total losses $=2662{\cdot}5$ watts.

Input $=V(I_a+I_{sh})=460\times52$ watts.

$=\dfrac{460\times52}{1000}=23{\cdot}92$ Kw.

Output $=\text{Input}-\text{losses}=23920-2662{\cdot}5$

$=21257{\cdot}5$ watts.

Efficiency $=\dfrac{\text{Output}}{\text{Input}}$

$=\dfrac{21257{\cdot}5}{23920}\times100=88{\cdot}8\%.$

H.P. output $=\dfrac{21257{\cdot}5}{746}=28{\cdot}5$ H.P.

Input current $=51+1=52$ amps.

Rating 28·5 H.P., 460 V, 52 amps., 940 rpm.

CHAPTER XIV

THE D.C. MOTOR

14-1. The electric motor is meant to convert electrical energy into mechanical energy. It depends for its action on the principle that when a conductor carrying current is placed across a magnetic field, it is acted upon by a force which tends to move it in a direction given by the Fleming's left hand rule.

Consider the magnetic field produced by the two poles N and S shown in the figure at (*a*). The conductor at (*b*) is shown with the current flowing away from the observer and having lines of force around it as shown. When this conductor is placed under the field produced by the magnets, a force acts on the conductor tending to move it to the left as represented by the arrow. The magnitude of this force is :

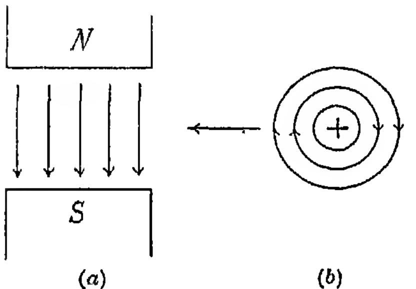


Fig. 101

$$f = \frac{HlI}{10} \text{ dynes.}$$

where H=Field strength in lines per sq. cm.

l=Length of conductor in cm.

I=Current in amps.

The conductors (carrying currents) on the armature of a motor experience such forces and it can be seen that all the forces acting on the conductors tend to turn the armature in the same direction.

14-2. Back Emf.

When the armature of a motor rotates the conductors on it cut the magnetic lines of force and an emf. is induced in these conductors. This induced emf. acts in opposition to the

applied voltage (and is therefore called Back emf.). This emf. is produced in the same way as the emf. in a generator. The applied voltage has to overcome this back emf. in addition to the drop.

14-3. Power output of a motor.

The current which flows in the armature is due to the voltage which is resultant of the applied voltage and the back emf.

i.e., Armature current $I_a = \dfrac{V-e}{R_a}$

where V=applied voltage

e=back emf.

R_a=Resistance of the armature circuit

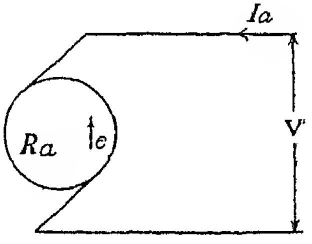


Fig. 102

Power supplied to the armature $= VI_a$

Power wasted

$$= I_a^2 R_a = I_a \times I_a \times R_a$$

$$= I_a \times \frac{V-e}{R_a} \times R_a$$

$$= I_a(V-e)$$

& watts. converted to mechanical power

$$= VI_a - I_a(V-e)$$

$$= eI_a \qquad \text{...Eq. (14-1)}$$

$=$Back emf. $\times$ Armature current.

However actual power available at the motor shaft is less than this mechanical power and is $= eI_a - P_1$ ...Eq. (14-2)

where P_1=Iron, friction and windage losses.

14-4. Torque (Useful Torque and Armature Torque).

If T is the torque available at the shaft of the motor and 'B.H.P' is the brake horse-power of the motor then

$$\text{B.H.P.} = \frac{2\pi NT}{33{,}000}$$

or

$$T = \frac{33{,}000 \times \text{B.H.P}}{2\pi N} \text{ lb. ft.}$$

where N is in r.p.m. Eq. (14-3)

This is the useful (or shaft) torque and is less than the armature torque, T_a. The loss in the torque is due to the iron and friction losses.

From Eq. (14-1) power available for mechanical conversion $= e\,Ia$ watts

$$= \frac{eIa}{746} \text{ H.P.} \qquad \ldots(i)$$

This is also equal to $\dfrac{2\pi NI_a}{33{,}000}$ $\qquad \ldots(ii)$

where N=speed of the armature in r.p.m.

T_a=armature torque in lb. ft.

Equating (*i*) and (*ii*)

$$\frac{eI_a}{746} = \frac{2\pi NT_a}{33{,}000}$$

or

$$T_a = \frac{33{,}000}{746} \times \frac{eI_a}{2\pi N}$$

Substituting the value of e

$$= \left(\frac{p\phi ZN}{10^8 q 60}\right)$$

$$T_a = \frac{33{,}000}{746 \times 2\pi} \times \frac{(p\phi ZN)I_a}{10^8 q 60 \times 2\pi N}$$

$$= \frac{33{,}000}{2\pi \times 746} \cdot K\phi I_a$$

where

$$K = \frac{pZ}{10^8 q 60}$$

$$= 7{\cdot}04 K\phi\, I_a \text{ lb. ft.} \qquad \ldots\text{Eq. (14-4).}$$

Eq. (14-4) shows that the armature torque (which would be approximately equal to the useful torque if iron and friction losses are neglected) is directly proportional to

(*i*) The flux

(*ii*) The armature current

or $\qquad T_a \propto \phi\, I_a$

or

$$\frac{T_{a1}}{T_{a2}} = \frac{\phi_1\, I_{a1}}{\phi_2\, I_{a2}} \qquad \ldots\text{Eq. (14-5)}$$

14-5. Relation between speed, back emf. and flux.

$$I_a = \frac{V - e}{R_a}$$

or
$$e = V - I_a R_a$$

or
$$\frac{p\phi ZN}{10^8 q 60} = V - I_a R_a$$

p, z, q, 10^8, 60 being constant,

$$N \propto \frac{V - I_a R_a}{\phi} \qquad \text{...Eq. (14-6)}$$

i.e.,
$$N \propto \frac{e}{\phi} \qquad \text{.. Eq. (14-7)}$$

Therefore the speed of a motor is directly proportional to the back emf. and inversely proportional to the flux.

i.e.,
$$\frac{N_1}{N_2} = \frac{e_1}{e_2} \times \frac{\phi_2}{\phi_1} \qquad \text{...Eq. (14-7}a\text{)}$$

The speed of a motor can, therefore, be varied by varying the applied voltage V, or resistance of the armature, or the flux ϕ. (*See* Eq. 14-6).

14-6. Motor Characteristics : There are three general types of motors-shunt, series and compound. They are named according to the way their fields are connected.

The suitability of a particular type of motor for a specific purpose can best be studied with the help of motor characteristics which are

(*i*) **Speed Load Characteristic :** The plot between the armature current (along x-axis) and speed (along y-axis) is called the speed load characteristic.

(*ii*) **Torque Load Characteristic :** The plot between the armature current (along x-axis) and torque (along y-axis) is called the torque load characteristic.

(*a*) **Shunt Motor Characteristic.**

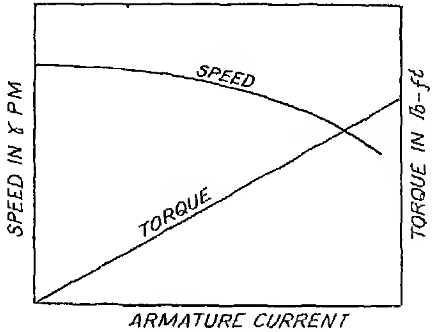


Fig. 103(*a*)
Curves showing the relation of torque and speed to the armature current in a shunt motor

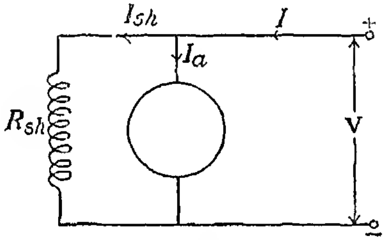


Fig. 103(*b*)

$$I_{sh} = \frac{V}{R_{sh}}.$$

$$I_a = I - I_{sh}$$

The speed of a shunt motor at full load falls below its speed at no load. However this drop in speed is so small that, so long as the load is not taken beyond the normal full load, the shunt motor is, to all intents and purposes, a constant-speed motor. If desired the speed at full load can be brought to its no load value by increasing the resistance in the field. (Increase in the field resistance decreases excitation and, therefore, increases the speed which is $\propto \frac{e}{\phi}$).

The torque of a shunt motor increases in direct proportion to the armature current since ϕ may be considered constant at all loads neglecting armature reaction.

Machine tools such as lathes, milling machines etc. require a motor which will maintain an efficient cutting speed under widely variable load conditions and this requirement is well satisfied by the shunt motor.

(b) Series Motor Characteristics.

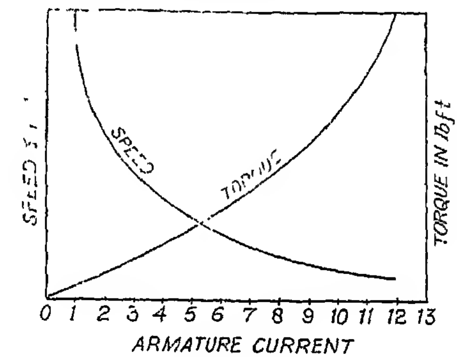


Fig. 104(a)

Curves showing the relation of torque and speed to the armature current in a series motor.

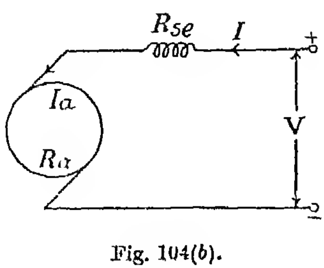


Fig. 104(b).

$$I_a = I.$$

$$e = V - I_a(R_a + R_{se}).$$

The no load speed of a series motor is too high to be plotted and the speed falls rapidly with the first increase of load and then more gradually as the load gets in the vicinity of full load.

Since there is racing on "no load" a series motor should never be run without load.

$T_a \propto I_a\phi$. At light loads, whilst working on the straight portion of the O.C.C., $\phi \propto I_a$.

$\therefore$ $T \propto I_a^2$: hence initial portion of torque characteristic is a parabola. At heavier loads when the iron is saturated the curve merges into a straight line as shown in fig 105(a). Thus the motor develops a very high torque at starting ; this torque decreasing as the speed increases. Such a characteristic is highly suitable for traction work and for cranes.

(c) Compound Motor Characteristic.

A compound motor may be

either. (i) **A Cumulative compounded motor :**

In this type of motor the shunt and the series excitations help one another. The motor has series characteristics ; does not race at no load ; gives large torques at low speeds. Such motors with flywheels are used with punch presses, power spears, rolling mills. Compound motors with weaker shunt field and stronger series field are used for driving power fans.

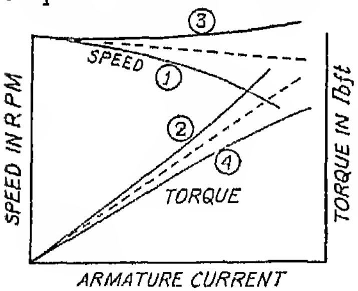


Fig. 105

Curves (1) & (2) are the speed & torque curves of cumulative compounded motor.

Curves 3 & 4 are the speed & torque curves of differential compounded motor.

Dotted ones are for the shunt motor.

or, (ii) **A differential compounded motor :** In this type of motor the two excitations oppose each other. The motor has exaggerated shunt characteristics ; maintains constant speed at all loads within limits. This type of motor finds limited application for experimental and research work.

Example 1. The armature of a 220 volt, 8 pole motor is wave wound with 888 conductors, the resistance of the

armature circuit being ·4 ohms. If the flux per pole is 1,200,000 lines, find the speed of the motor when taking an armature current of 60 amps.

$$\text{Back emf. } e=220-60\times{\cdot}4=196 \text{ V.}$$

$$e=\frac{p\phi ZN}{10^8 q\times 60}$$

$$\text{or } 196=\frac{8\times 1{\cdot}2\times 10^6\times 888N}{10^8\times 2\times 60}$$

$$\therefore N=\frac{1500\times 196}{1{\cdot}2\times 888}=276 \text{ rpm.} \quad \textit{Ans.}$$

Example 2. A 500 V. shunt motor has 6 poles and a wave wound armature with 1200 conductors The useful magnetic flux per pole is 2 million lines and the armature and field resistances are ·5 and 250 ohms. respectively. Ignoring armature reaction find the speed at which it will run when taking 20 amps. from the mains.

$$\text{Shunt field current} =\frac{500}{250}=2 \text{ amps.}$$

$$\text{Armature current} =20-2=18 \text{ amps.}$$

$$e=500-18\times{\cdot}5=491 \text{ volts.}$$

$$491=\frac{6\times 2\times 10^6\times 1200N}{10^8\times 2\times 60}$$

$$N=409{\cdot}2 \text{ rpm.} \quad \textit{Ans.}$$

Example 3. A 4 pole shunt motor has 720 conductors on its lap wound armature and a flux of 4 megalines per pole. The armature resistance is ·1 ohm and the brush contact drop is one volt per brush. Find the speed of the machine when it takes an armature current of 40 amps from the 500 V. supply.

There are 4 poles and 4 brushes.

Brush drop $=1+1=2$ volts. $\because$ 2 sets are in parallel.

Back emf. $e=500-2-I_a R_a$

$$=500-2-40\times{\cdot}1=494 \text{ volts.}$$

$$e=\frac{p\phi ZN}{10^8\times q\times 60}$$

$$494 = \frac{4 \times 4 \times 10^6 \times 720N}{10^8 \times 4 \times 60} = \frac{48N}{100}$$

$$N = \frac{494 \times 100}{48} = 1029 \text{ rpm.} \quad \textit{Ans.}$$

Example 4. A 4 pole 440 volt 80 H.P. motor has a full load efficiency of 90%. The armature is wave wound with 468 conductors. Total resistance of armature including the interpoles is ·1 ohm, shunt field resistance is 220 ohms. Flux per pole is 4 megalines. Calculate the full load speed and torque in lb. ft.

Motor input at full load

$$= 80 \times 746 \times \frac{100}{90} \text{ watts.}$$

Line current at full load

$$= 80 \times 746 \times \frac{100}{90} \times \frac{1}{440} = 150{\cdot}7 \text{ amps.}$$

Field current $= \frac{440}{220} = 2$ amps.

Armature current $= 150{\cdot}7 - 2 = 148{\cdot}7$ amps.

Back emf. $e = 440 - 148{\cdot}7 \times {\cdot}1 = 425{\cdot}13$ volts

$$425{\cdot}13 = \frac{4 \times 4 \times 10^6 \times 468N}{10^8 \times 2 \times 60} = \frac{2 \times 468N}{1500}$$

$$N = \frac{1500 \times 425{\cdot}13}{2 \times 468} = 681 \text{ rpm.}$$

Useful torque $= \frac{HP \times 33000}{2\pi N} = \frac{80 \times 33000 \times 7}{2 \times 22 \times 681}$

$= 616{\cdot}7$ lb. ft. *Ans.*

Example 5. A shunt motor has an armature resistance of ·5 ohm. and takes a no load current of 5 amps. at 220 volts when running at a speed of 600 rpm. If the field current is constant at one amp., determine the speed of the machine when taking an armature current of 30 amps. Ignore commutator drop.

Armature current at no load

$$=5-1=4 \text{ amps.}$$

Back emf. e_1 at no load

$$=220-I_aR_a=220-4\times\cdot5$$
$$=218 \text{ volts.}$$

When loaded the armature current

$$=30 \text{ amps.}$$

Back emf. on load $=e_2=220-30\times\cdot5=205$ volts.

$$\frac{e_1}{e_2}=\frac{N_1}{N_2}$$

$$N_2=600\times\frac{205}{218}=564 \text{ rpm.} \quad \textit{Ans.}$$

Example 6. A shunt generator has an output of 20 Kw. at 250 volts and 500 rpm. The armature resistance is ·1 ohm and the field resistance 125 ohms. Calculate its speed as a shunt motor when taking 20 Kw. from 250 V. mains.

As a Generator.

Current supplied to outside circuit

$$=\frac{20\times100}{250}=80 \text{ amps.}$$

Field current $=\frac{250}{125}=2$ amps.

Armature current $=80+2=82$ amps.

Armature drop $=82\times\cdot1=8\cdot2$ volts.

Induced emf. $E=250+8\cdot2=258\cdot2$ volts.

Speed $N_1=500$ rpm.

As a Motor.

Current taken from the line

$$=\frac{20\times1000}{250}=80 \text{ amps.}$$

Field current $=\frac{250}{125}=2$ amps.

Armature current $=80-2=78$ amps.

Back emf. $e=250-78\times\cdot 1$

$=242{\cdot}2$ volts.

Speed $N_2=N_1\times\frac{e}{E}=\frac{500\times 242{\cdot}2}{258{\cdot}2}$

$=469$ rpm. *Ans.*

Example 7. A 4 pole D C. shunt motor runs at 1000 rpm. on a 460 V. supply when taking an armature current of 45 amps., the resistance of the armature circuit being ·4 ohm. Find the torque developed by the machine in lb. ft. If the field circuit supply remains 460 volts while the armature is supplied with only 60 volts and is to develop the same torque, find the speed under these conditions.

Back emf. $e=V-I_aR_a=460-45\times\cdot 4=442$ V.

Power developed by the motor

$=442\times I_a=442\times 45$ watts.

If T is the torque developed by the motor in lb. ft.; then

$$\frac{2\pi NT}{33000}\times 746=442\times 45$$

or
$$T=\frac{442\times 45\times 33000}{2\pi\times 1000\times 746}=140 \text{ lb. ft.}$$

Speed when the armature is supplied with only 60 V. and the motor is to exert the same torque *i.e.*, a torque of 140 lb. ft.

Torque $\propto\phi I_a=$ constant,

and since ϕ is constant, therefore I_a is also constant.

$\therefore$ $I_a=45$ amps.

Back emf. $e_1=V_1-I_aR_a=60-45\times\cdot 4$

$=42$ volts.

$$\frac{2\pi N_1\times 140\times 746}{33000}=42\times 45$$

where N_1 is the speed in this case

$$\therefore \quad N_1=\frac{42\times 45\times 33000}{2\pi\times 140\times 746}=95 \text{ rpm.} \quad \textit{Ans.}$$

Example 8. A 440 V. D.C. shunt motor has a field resistance (cold) of 100 ohms and an armature resistance of ·04 ohm (cold). Its no load speed when cold is 1200 rpm. After running on load for some time both armature and field have a mean temp. rise above the atmosphere of 35°C. If the armature current of the motor is 90 amps., calculate its speed. Temp. coefficient of resistance= ·0042. Assume that the flux changes in the same ratio as the field current.

The armature current at no load being very small the armature drop can be neglected and the back emf. at no load

$$e_0=440 \text{ V.}$$

$$N_0=1200 \text{ rpm.}$$

$$\phi_0 \propto \frac{440}{100}$$

With the increased temp.

$$R_{sh}=100(1+\cdot0042\times35)=114\cdot7 \text{ ohms.}$$

$$R_a=\cdot04\times1\cdot147=\cdot046 \text{ ohm.}$$

Back emf. e now $=440-90\times\cdot046=440-4\cdot16$

$$=435\cdot84 \text{ V.}$$

Field current $=\dfrac{440}{114\cdot7}$

The flux now $\phi=\phi_0\times\dfrac{440}{114\cdot7}\times\dfrac{100}{440}$

$$=\phi_0\times\frac{100}{114\cdot7}$$

$$e_0=\frac{p\phi_0ZN_0}{10^8q60}$$

$$e=\frac{p\phi ZN}{10^8q60}$$

where N=speed at the higher temp.

$$\frac{e}{e_0}=\frac{p\phi ZN}{p\phi_0ZN_0}=\frac{\phi N}{\phi_0N_0}$$

But $$\phi=\frac{100}{114\cdot7}\phi_0$$

$$\frac{e}{e_0}=\frac{100}{114\cdot7}\,\frac{\phi_0 N}{\phi_0 N_0}=\frac{100}{114\cdot7}\times\frac{N}{N_0}$$

$$\frac{435\cdot84}{440}=\frac{100N}{114\cdot7N_0}$$

$$N=\frac{435\ 84\times114\ 7\times1200}{440\times100}$$

$$=1363 \text{ rpm.} \quad \textit{Ans.}$$

Example 9. A 250 volt D.C. shunt motor has an armature circuit resistance of ·5 ohm and the field circuit resistance of 125 ohms. It drives a load at 1000 rpm. taking a current of 30 amps. from the line. The field circuit resistance is slowly raised to 150 ohms. If the flux is proportional to field current and if the load torque is constant find the final speed and the armature current.

$$I_{sh1}=\frac{250}{125}=2 \text{ amps.}$$

$$I_{a1}=30-2=28 \text{ amps.}$$

$$\text{Back emf. } e_1=250-28\times\cdot5=236 \text{ volts.}$$

$$N_1=1000 \text{ rpm.}$$

ϕ_1 is proportional to 2 amps.

When field circuit resistance becomes 150 ohms.

$$I_{sh2}=\frac{250}{150}=1\cdot67 \text{ amps.}$$

Torque $\propto$ Flux $\times$ Armature current.

Since torque is constant

$$\phi_1 I_{a1}=\phi_2\ I_{a2}$$

$$I_{a2}=\frac{\phi_1\times I_{a1}}{\phi_2}=\frac{I_{sh1}}{I_{sh2}}\times I_{a1}=\frac{2\times3\times28}{5}$$

$$=33\cdot6 \text{ amps.}$$

Back emf. in the second case $= e_2$

$$= 250 - 33{\cdot}6 \times {\cdot}5 = 233{\cdot}2 \text{ volts.}$$

$$\frac{e_1}{e_2} = \frac{\phi_1 N_1}{\phi_2 N_2} = \frac{2 \times 3}{5} \times \frac{1000}{N_2}$$

$$\frac{236}{233{\cdot}2} = \frac{2 \times 3}{5} \times \frac{1000}{N_2}$$

$$N_2 = \frac{6}{5} \times \frac{1000 \times 233{\cdot}2}{236} = 1186 \text{ rpm. } \textit{Ans.}$$

Example 10. A shunt motor runs at 600 rpm. on a 240 V. supply. Its armature and field resistances are ·5 and 120 ohms. respectively and the total current taken from the supply is 32 amps.

(*a*) What resistance should be placed in series with the armature circuit to reduce the speed to 400 rpm. with no change in the armature or field current.

(*b*) In what ratio is the H P. reduced.

(*c*) To what value should the load be changed so that with the added resistance the speed comes to its former value of 600 rpm. Ignore the effect of armature reaction.

a) Field current $= \frac{240}{120} = 2$ amps.

$$I_a = 32 - 2 = 30 \text{ amps.}$$

Back emf. e_1 $= 240 - {\cdot}5 \times 30 = 225$ volts.

At 400 rpm. the back emf. e_2

$$= 225 \times \frac{400}{600} = 150 \text{ volt.}$$

The voltage $e_1 - e_2$ must be absorbed in the external resistance put in series with the armature

$$e_1 - e_2 = 225 - 150 = 75 \text{ volts.}$$

Resistance $= \frac{75}{30} = 2{\cdot}5$ ohms.

(*b*) Power developed by a motor is proportional to back emf. *See* Eq. (14-1).

So H P is reduced to $\frac{150}{225}$ or $\frac{2}{3}$ of its former value.

(c) The speed shall be 600 rpm. with the added resistance in the circuit if the back emf is equal to e_1, *i.e.* 225 volts

Armature drop $=240-225=15$ volts.

Armature circuit resistance

$$=2{\cdot}5+{\cdot}5$$
$$=3 \text{ ohms.}$$

Armature current $=\frac{15}{3}=5$ amps.

Total current taken by the motor

$$=5+2=7 \text{ amps.} \quad \textit{Ans.}$$

Example 11. A 4 pole 220 V. shunt motor having 540 lap wound conductors, takes a current of 32 amps. from the line and develops 7·5 H.P. The field current is 2 amps. and the flux per pole is 3 megalines. The armature resistance is ·8 ohms. Calculate the total torque developed and the useful torque.

Armature current $I_a=32-2=30$ amps.

Back emf. $e=220-30\times{\cdot}8\times=196$ volts.

$$e=\frac{p\phi Z}{10^8 q}\frac{N}{60}$$

$$196=\frac{4\times3\times10^6\times540N}{10^8\times4\times60}=\frac{3\times540N}{100\times60}$$

$$N=\frac{19600}{27}$$

Total torque developed in lb. ft.

$$=7{\cdot}04\times\frac{eI_a}{N}$$

$$=\frac{7{\cdot}04\times196\times30\times27}{19600}$$

$$=57{\cdot}024 \text{ lb. ft.}$$

Useful torque $=\frac{HP\times33000}{2\pi N}$

$$=\frac{7{\cdot}5\times33000\times27\times7}{2\times22\times19600}=\frac{7{\cdot}5\times30\times27}{4\times28}$$

$$=54{\cdot}24 \text{ lb. ft.} \quad \textit{Ans.}$$

Example 12. A 100 V. series motor has an armature resistance of ·2 ohm and field resistance of ·25 ohm. When its pulley exerts a torque of 20 lb. ft. it runs at a speed of 600 rpm. Iron, friction losses at this speed are 300 watts. Calculate lost torque, copper losses and efficiency.

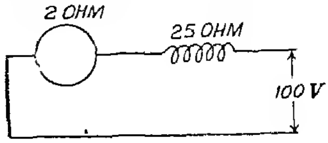


Fig. 106

If lost torque is T_l lb. ft. we have

$$\frac{2\pi N T_l \times 746}{33000} = 300$$

so that $$T_l = \frac{300 \times 33000}{2\pi \times 600 \times 746} = 7{\cdot}04 \times \frac{300}{600}$$
$$= 3{\cdot}52 \text{ lb. ft.}$$

Torque developed $= 20 + 3{\cdot}52 = 23{\cdot}52$ lb. ft.

If I_a is the armature current we have

Back emf. $e = 100 - {\cdot}45\, I_a$

Torque developed T $= \dfrac{eI_a}{N} \times 7{\cdot}04$

$$23{\cdot}52 = \frac{(100 - {\cdot}45\, I_a) \times I_a \times 7{\cdot}04}{600}$$

or $$\frac{23{\cdot}52 \times 600}{7{\cdot}04} = 100 I_a - {\cdot}45 I_a^2$$

or $${\cdot}45 I_a^2 - 100 I_a + 2004 = 0$$

$$I_a = \frac{100 \pm \sqrt{100^2 - 4 \times {\cdot}45 \times 2004}}{2 \times {\cdot}45}$$
$$= 22{\cdot}25 \text{ amps.}$$

Copper loss $= 22{\cdot}25^2 \times {\cdot}45 = 222{\cdot}75$ watts.

Total losses $= 222{\cdot}75 + 300 = 522{\cdot}75$ watts.

Input $= 100 \times 22{\cdot}25 = 2225$ watts.

Output $= 2225 - 522{\cdot}25 = 1702{\cdot}25$

Efficiency $= \dfrac{1702{\cdot}25}{2225} = 76{\cdot}5\%$ *Ans.*

Example 13. A 460 V. series motor runs at 500 rpm. taking a current of 40 amps. Find the speed and percentage change in torque if the load is reduced so that the current is reduced to 30 amps. Total resistance of armature and field circuit is ·8 ohm. Assume that the flux is proportional to the field current.

When motor is taking 40 amps. the back emf.

$$e_1 = 460 - 40 \times \cdot 8 = 428 \text{ volts.}$$

When taking 30 amps. the back emf.

$$e_2 = 460 - 30 \times \cdot 8 = 436 \text{ volts.}$$

$$\frac{e_1}{e_2} = \frac{\phi_1 N_1}{\phi_2 N_2}$$

Flux being proportional to current

$$\frac{e_1}{e_2} = \frac{40 \times 500}{30 \times N_2}$$

or
$$\frac{428}{436} = \frac{40 \times 500}{30 N_2}$$

or
$$N_2 = 680 \text{ rpm.}$$

Torque is proportional to Flux × Armature current

$$\frac{T_1}{T_2} = \frac{\phi_1 I_{a1}}{\phi_2 I_{a2}} = \frac{40}{30} \times \frac{40}{30} = \frac{16}{9}$$

$$T_2 = \frac{9}{16} T_1$$

Percentage change of torque

$$= \frac{T_1 - T_2}{T_1} \times 100$$

$$= 43 \cdot 75 \quad \textit{Ans.}$$

Example 14. The full load armature current of a 440 V. shunt motor is 120 amps. The armature resistance is 0·2 ohm and speed 800 rpm. What will be the speed if the torque is reduced to 60% of its full load value and a resistance of 1·5 ohm. is connected in series with the armature circuit. The field strength is the same in both cases.

Back emf. $e_1 \qquad = 440 - 120 \times \cdot 2 = 416$ volts.

$$N_1 = 800 \text{ rpm.}$$

Let T_1 be the full load torque and T_2 the torque in the second case.

$$\frac{T_2}{T_1} = \cdot 6$$

$$\frac{T_1}{T_2} = \frac{\phi_1 \times I_{a1}}{\phi_2 \times I_{a2}}$$

As the flux in both cases is equal

$$\therefore \qquad \frac{T_1}{T_2} = \frac{I_{a1}}{I_{a2}}$$

or $$I_{a2} = I_{a1} \times \frac{T_2}{T_1}$$

$$= 120 \times \cdot 6 = 72 \text{ amps.}$$

$$e_2 = 440 - 72(\cdot 2 + 1 \cdot 5) = 440 - 122 \cdot 4$$

$$= 317 \cdot 6 \text{ volts.}$$

$$\frac{e_1}{e_2} = \frac{N_1}{N_2}$$

or $$N_2 = N_1 \times \frac{e_2}{e_1}$$

$$N_2 = 800 \times \frac{317 \cdot 6}{416} = 610 \text{ rpm.} \qquad \textit{Ans.}$$

Example 15. A 200 V. shunt motor when running light takes a current of 6 amps, the speed being 1200 rpm., armature resistance ·15 ohm, shunt field resistance 125 ohms. A series winding of 0·05 ohm resistance is added being connected long shunt cumulative. The series winding increases the flux per pole by 25% when the motor takes its full load current of 101·6 amps. Ignoring armature reaction find (*a*) the speed of the motor when taking full load current as a compound motor. (*b*) compare the torque exerted with and without the compound winding.

Note. Cumulative compounding means that the series field winding is so connected that its flux assists the shunt field flux.

When running light.

Field Current $=\frac{200}{125}=1{\cdot}6$ amps.

Armature Current $=6-1{\cdot}6=4{\cdot}4$ amps.

$e_1=200-4{\cdot}4\times{\cdot}15=199{\cdot}34$ volts.

When the series winding is connected.

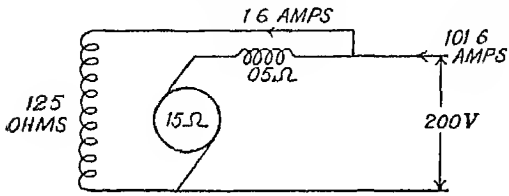


Fig. 107

Shunt field current $=1{\cdot}6$ amps.

$I_a=101{\cdot}6-1{\cdot}6$
$=100$ amps.

Volts drop in armature and series field $=100\times{\cdot}2$
$=20$ volts.

$e_2=200-20=180$ volts.

$$\frac{e_1}{e_2}=\frac{N_1\times\phi_1}{N_2\times\phi_2}$$

or $$\frac{199{\cdot}34}{180}=\frac{1200\phi_1}{N_2\times1{\cdot}25\ \phi_1}$$

or $$N_2=\frac{1200\times180}{1{\cdot}25\times199{\cdot}34}=866\text{ rpm.}$$

Torque is proportional to $I_a\times\phi$.

With the series winding connected the flux is $1{\cdot}25\times$flux without series winding and so is the torque. (Armature current is 100 amps. in both cases).

Example 16. A D.C. series motor runs at 1000 rpm. when taking 20 amps. at 200 volts. The resistance of the armature is ·5 ohm and that of the field winding 0·2 ohm. Find the speed for a total current of 20 amps. when a 0·2 ohm. resistance is connected across the series winding. The flux for a field current of 10 amps. is 70% of that for 20 amps.

In the first case

Back emf e_1 $=200-20\times({\cdot}5+{\cdot}2)=186$ volts.

In the second case with ·2 ohm resistance connected across the series field.

Field current = 10 amps.

$I_a = 20$ amps.

$e_2 = 200 - 10 \times \cdot 2 - 20 \times \cdot 5$

$= 200 - 2 - 10$

$= 188$ volts.

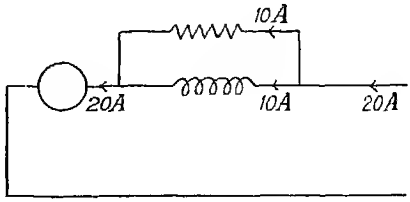


Fig. 108

$$\frac{e_1}{e_2} = \frac{N_1 \phi_1}{N_2 \phi_2}$$

or $$\frac{186}{188} = \frac{1000 \times \phi_1}{N_2 \times \cdot 7 \phi_1}$$

$$[N_2 = \frac{188 \times 1000}{186 \times \cdot 7} = 1444 \text{ rpm. } \textit{Ans.}$$

Example 17. The resistance of each of the two field coils of a series motor is ·04 ohm and armature resistance 04 ohm. The machine takes a current of 50 amps. at 100 volts and runs at 800 rpm, when the two field coils are connected in series.

Find the speed when the two coils are connected in parallel and the load torque is (*a*) doubled (*b*) halved. Assume that the flux is proportional to the field current.

When the coils are in series :

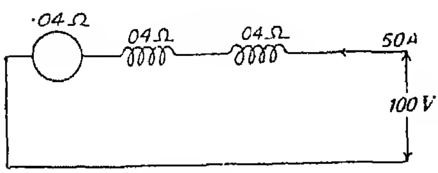


Fig. 109

Total resistance

$= \cdot 04 \times 3$

$= \cdot 12$ ohm.

Back emf. e_1

$= 100 - 50 \times \cdot 12$

$= 94$ volts.

$N_1 = 800$ rpm.

When field coils are in parallel and torque is doubled.

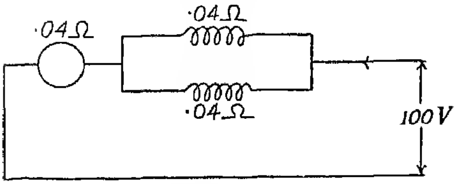


Fig. 110

$$\frac{T_1}{T_2}=\frac{I_{a1}\times\phi_1}{I_{a2}\times\phi_2}$$

$$\frac{1}{2}=\frac{50\times 50}{I_{a2}\times\frac{I_{a2}}{2}}$$

$$I_{a2}\times\frac{I_{a2}}{2}=50\times 50\times 2$$

$$(I_{a2})^2=50\times 50\times 4$$

$$I_{a2}=100 \text{ amps.}$$

Back emf. e_2 $=100-100\times{\cdot}04-50\times{\cdot}04$

$=100-4-2=94$ volts.

$$\frac{e_1}{e_2}=\frac{N_1\phi_1}{N_2\phi_2}$$

or $$\frac{94}{94}=\frac{800\times 50}{50\ N_2}$$

$$N_2=800 \text{ rpm.}$$

When torque is halved :

$$\frac{T_1}{T_2}=2=\frac{I_{a1}\phi_1}{I_{a2}\phi_2}=\frac{50\times 50}{I_{a2}\times\frac{I_{a2}}{2}}$$

$$I_{a2}=50 \text{ amps.}$$

$$e_2=100-50\times{\cdot}04-25\times{\cdot}04$$

$$=100-2-1=97 \text{ volts.}$$

$$\frac{e_1}{e_2}=\frac{N_1\phi_1}{N_2\phi_2}$$

or $$\frac{94}{97}=\frac{800\times 50}{N_2\times 25}$$

$$N_2=\frac{800\times 50\times 97}{25\times 94}$$

$=1650$ rpm. *Ans.*

Example 18. A 250V series motor having a resistance of ·5 ohm runs at 800 rpm. when taking a current of 40 amps.

Find the speed when the current is 25 amps. if the flux at this value of current is 25% less than the flux at 40 amps.

At 40 amps. current the back emf.

$$e_1 = 250 - 40 \times \cdot 5 = 230 \text{ volts.}$$

At 25 amps. $\quad e_2 = 250 - 25 \times \cdot 5 = 237 \cdot 5$ volts.

Speed N_2 if flux had remained the same

$$= 800 \times \frac{237 \cdot 5}{230}$$

When the flux is 75% of the former value

Speed $\quad = 800 \times \frac{237 \cdot 5}{230} \times \frac{100}{75}$

$$= 1101 \text{ rpm.} \quad \textit{Ans.}$$

Example 19. Two exactly similar motors are operating in parallel and each is taking the same armature current while the field strength of one is made 75% of the other. Compare torques, speeds and outputs of the motors.

As the armature currents are equal, the back emfs. e_1 and e_2 are equal.

$$\phi_1 N_1 = \phi_2 N_2$$

If $\quad \phi_2 = \cdot 75\ \phi_1$ then $\phi_1 N_1 = \cdot 75\ \phi_1 N_2$

$$\frac{N_2}{N_1} = \frac{1}{\cdot 75}$$

Let T_1 and T_2 be the respective torques

$$T_1 = KI\phi_1$$

$$T_2 = KI\phi_2 = KI \times \cdot 75\ \phi_1$$

$$\frac{T_2}{T_1} = \frac{\cdot 75 \phi_1}{\phi_1} = \cdot 75$$

Let outputs be W_1 and W_2

$$W_1 \propto T_1 N_1$$

$$W_2 \propto T_2 N_2$$

$$\frac{W_2}{W_1} = \frac{T_2 N_2}{T_1 N_1} = \frac{\cdot 8}{\cdot 8} = 1$$

or the outputs are equal. *Ans.*

Example 20. A 4 pole series wound fan motor runs normally at 600 rpm. on a 25 volt supply taking a current of 20 amps. The field coils are all connected in series. Estimate the speed and the current taken by the motor if the coils are con-

nected in two parallel groups of 2 coils in series. The load torque increases as the square of the speed. Assume that the flux is directly proportional to the current and ignore losses.

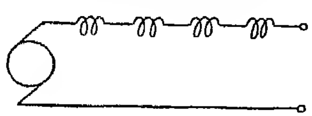


Fig. 111 (*a*)

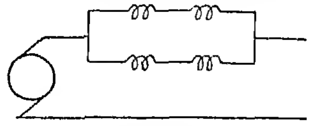


Fig. 111 (*b*)

$I_{sh1} = I_{a1}$ $\qquad\qquad$ $I_{sh2} = \frac{I_{a2}}{2}$

Speed $N = \frac{KV}{I_{sh}} = K\frac{V}{I_a}$ because the flux is proportional to current and the losses are neglected.

Torque $T \propto I_a \phi$ or $I_{sh} \phi$

$$N_1 = K_1 \frac{V}{I_{a1}}$$

$$N_2 = K_2 \frac{V}{I_{a2}} \cdot K_1 \times \frac{2V}{I_{a2}}$$

$$\frac{N_1}{N_2} = \frac{V}{I_{a1}} \times \frac{I_{a5}}{2V} = \frac{I_{a2}}{2I_{a1}} \qquad \ldots(1)$$

$$T_1 = aN_1^2 = b\, I_{a1} \times I_{sh1} = b\, I_{a1}^2$$

$$T_2 = aN_2^2 = b\, I_{a2} \times I_{sh2} = \frac{b}{2}\, I^2_{a2}$$

where a and b are constants.

$$\frac{T_1}{T_2} = \frac{N_1^2}{N_2^2} = \frac{2I_{a1}^2}{I_{a2}^2}$$

$$\frac{N_1}{N_2} = \sqrt{2}\, \frac{I_{a1}}{I_{a2}} \qquad \ldots(2)$$

From equations (1) and (2)

$$\frac{I_{a2}}{2I_{a1}} = \frac{\sqrt{2}\, I_{a1}}{I_{a2}}$$

$$I_{a2} = \sqrt{2 \times \sqrt{2}}\, I_{a1}$$

$$= \sqrt{2 \times \sqrt{2}} \times 20 = 33{\cdot}5 \text{ amps.}$$

$$N_2 = \frac{2I_{a1}\, N_1}{I_{a2}} = \frac{40 \times 600}{33{\cdot}5} = 716 \text{ rpm.}$$

Ans.

CHAPTER XV

LOSSES AND EFFICIENCY OF D.C. MOTORS

15-1. The losses occurring in a D.C. generator have been discussed in Chapter XIII. Similar losses occur in the case of a D.C. motor. A generator converts mechanical power into electrical power but a motor converts electrical power into mechanical power.

The total electrical power VI supplied to the motor is utilised in two ways :

(*a*) To supply the copper losses in the field magnets and the armature.

(*b*) To force the armature current I_a in opposition to the induced emf e.

The part (*b*) equals eI_a and is the actual mechanical power developed by the armature. A part of this mechanical power developed by the armature is spent in friction, windage and iron losses and the remaining is delivered as useful mechanical power at the pulley. This can be represented by a diagram.

A			B		C
Electrical Power Input VI	−	Copper losses =	Mechanical Power— developed by armature eI_a	Iron, friction windage losses	Mechanical output

A=Line voltage × current taken by motor
=VI watts.

B - Armature emf $\times I_a = eI_a$ watts.

C=BHP output × 746 watts.

Commercial or overall efficiency

$$=\frac{C}{A}=\frac{eI_a-W}{VI}$$

where W = Iron, friction and windage losses.

$$=\frac{\text{Mechanical output}}{\text{Electrical Input}}$$

Electrical efficiency $=\frac{B}{A}=\frac{\text{Electrical power developed}}{\text{Electrical Input}}$

$$=\frac{eI_a}{VI}$$

Mechanical Efficiency $=\frac{C}{B}=\frac{\text{Mechanical output}}{\text{Electrical power developed}}$

$$=\frac{eI_a-W}{eI_a}$$

Here, as in the case of generators, unless otherwise stated, the term efficiency is understood to mean overall efficiency.

15·2. Maximum Efficiency.

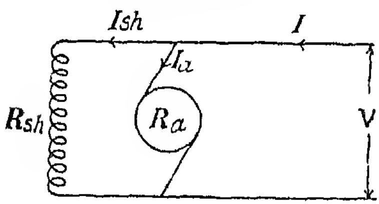


Fig. 112

As discussed earlier maxm. efficiency occurs when constant losses = variable losses.

Let us take the case of a shunt motor.

Variable losses $=I_a^2 R_a$

$=I^2R_a$ (approx.)

Let Constant losses = Shunt field loss + Iron, friction and windage losses

$=P$

Efficiency $\quad \eta=\frac{\text{Input}-\text{Losses}}{\text{Input}}$

$$=\frac{VI-P-I^2R_a}{VI}$$

$$\frac{d\eta}{dI}=\frac{(VI-P-I^2R_a)V-VI(V-2IR_a)}{V^2I^2}$$

For maximum efficiency

$$\frac{d\eta}{dI}=0$$

or $\quad VI-P-I^2R_a \qquad =VI-2I^2R_a$

or $\qquad\qquad I^2R_a=P'$

This shows that efficiency is a maximum when constant losses=variable losses.

Example 1. A series motor takes a current of 25 amps. at 220 volts The armature and field resistances are 2 and ·15 ohms. respectively and stray power loss is 700 watts. Find the electrical, mechanical and overall efficiency of the machine.

$$\text{Back emf. } e \qquad =220-25(\cdot 2+\cdot 15)$$
$$=220-8\ 75$$
$$=211\cdot 25 \text{ volts.}$$

$$\text{Electrical Efficiency} \qquad =\frac{eI_a}{VI}$$

In this case $I_a \qquad = I$

∴ Electrical Efficiency

$$=\frac{e}{V}=\frac{211\cdot 25}{220}$$
$$=\cdot 96=96\%.$$

$$\text{Mechanical Efficiency}=\frac{eI_a-W}{eI_a}$$
$$=\frac{211\cdot 25\times 25-700}{211\cdot 25\times 25}$$
$$=\frac{5281-700}{5281}=\cdot 867$$
$$=86\cdot 7\%$$

Overall efficiency $\qquad$ =Electrical efficiency × Mech
ci

$$=\frac{eI_a-W}{VI}=\frac{5281-700}{220\times 25}=\frac{45}{2.0}$$
$$=\cdot 833=83\cdot 3\%. \quad \textit{Ans}$$

Example 2. A shunt motor takes a current of 24 amps. at 220 volts. Calculate the various efficiencies given that its armature resistance is ·2 ohm, shunt field resistance 55 ohms, iron, friction and windage loss at the given speed is 700 watts.

$$\text{Shunt field current} \quad =\frac{220}{55}=4 \text{ amps.}$$

$$\text{Armature current} \quad =24-4=20 \text{ amps.}$$

$$\text{Armature drop} \quad =20\times{\cdot}2=4 \text{ volts.}$$

$$\text{Back emf. } e \quad =220-4=216 \text{ volts.}$$

$$\text{Electrical efficiency} \quad =\frac{eI_a}{VI}=\frac{216\times20}{220\times24}$$

$$={\cdot}818=81{\cdot}8\%$$

$$\text{Mechanical efficiency} =\frac{eI_a-W}{eI_a}=\frac{216\times20-700}{216\times20}$$

$$=\frac{3620}{4320}={\cdot}837=83{\cdot}7\%$$

$$\text{Overall efficiency} \quad =\frac{eI_a-W}{VI}=\frac{3620}{220\times24}\ 1{\cdot}685=68{\cdot}5\%.$$

Example 3. A long shunt compound motor takes a current of 24 amps. from 220 volt mains. Calculate the various efficiencies being given

$R_a={\cdot}1$ ohm, Resistance of series field$={\cdot}08$ ohm, Resistance of shunt field$=55$ohms, W at the given speed and voltage $=700$ watts.

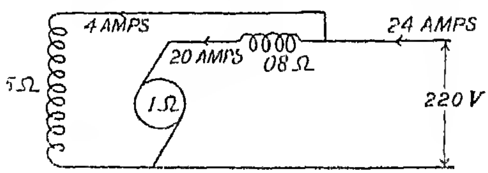


Fig. 113

$$\text{Shunt field current} \quad \frac{220}{55}=4 \text{ amps.}$$

$$\text{Armature current} \quad 24-4=20 \text{ amps.}$$

$$\text{Back emf. } e \quad =220-20({\cdot}1+{\cdot}08)$$

$$=220-3{\cdot}6=216{\cdot}4 \text{ volts.}$$

Electrical efficiency $=\dfrac{eI_a}{VI}=\dfrac{216{\cdot}4\times20}{220\times24}={\cdot}819$

$=81{\cdot}9\%$

Mechanical efficiency $=\dfrac{eI_a-W}{eI_a}=\dfrac{216{\cdot}4\times20-700}{216{\cdot}4\times20}$

$=\dfrac{3628}{4328}={\cdot}838=83{\cdot}8\%$

Overall efficiency $=\dfrac{eI_a-W}{VI}=\dfrac{3628}{220\times24}={\cdot}689$

$=68{\cdot}9\%$. *Ans.*

Example 4. Find the efficiency of the motor of example 3 when connected short shunt compound.

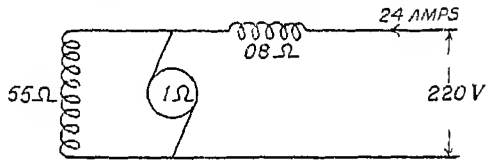


Fig. 114

Voltage drop in the series field

$=24\times{\cdot}08=1{\cdot}92$ volts.

Voltage across the shunt field

$=220-1{\cdot}92=218{\cdot}08$ volts.

Shunt field current $=\dfrac{218{\cdot}08}{55}=3{\cdot}96$ amp.

Armature current $=24-3{\cdot}96=20{\cdot}04$ amps.

Armature drop $=20{\cdot}04\times{\cdot}1=2{\cdot}004$ volts.

Back emf. e $=218{\cdot}08-2{\cdot}004=216{\cdot}07$ volts.

Electrical efficiency $=\dfrac{eI_a}{VI}=\dfrac{216{\cdot}07\times20{\cdot}04}{220\times24}$

$={\cdot}82=82\%$

Mechanical efficiency $=\dfrac{eI_a-700}{eI_a}=\dfrac{4328-700}{4328}$

$={\cdot}838=83{\cdot}8\%$

Overall efficiency $= \dfrac{eI_a - 700}{VI} = \dfrac{3628}{220 \times 24} = \cdot 689$

$= 68 \cdot 9\%$. *Ans.*

Example 5. A shunt motor running a centrifugal pump takes a current of 30 amps. from 220V mains at full load. The field current is one amp. When disconnected from the pump and run light it takes an armature current of 2·5 amps. If the armature resistance is ·25 ohm what power did the motor supply to the pump.

No load losses (*i.e.*, Iron friction and windage losses) $= W$

$= 2 \cdot 5 \times 220 = 550$ watts.

Full load armature current

$= 30 - 1 = 29$ amps.

Full load input to the armature

$= VI_a = 220 \times 29$ watts.

Losses $= I_a^2 R_a + W$

$= 29^2 \times \cdot 25 + 550 = 210 + 550$

$= 760$ watts.

Power supplied to the pump

$=$ Input $-$ Losses

$= 220 \times 29 - 760$ watts.

$= 6380 - 760 = 5620$ watts. *Ans.*

Example 6. A 20 H.P. 220 volt 1000 rpm. shunt motor has an efficiency of 90%, an armature resistance of ·075 ohm and a shunt field current of 2 amps. If the speed of the machine is reduced to 500 rpm. by introducing a resistance in the armature circuit, the torque of the load remaining the same find (*a*) the output of the motor, (*b*) the armature current, (*c*) the value of the external resistance and (*d*) the overall efficiency of the machine.

At load of 20 H.P. the motor input

$= \dfrac{20}{\cdot 9} = 22 \cdot 22$ H.P. $= 16570$ watts.

Current input to the motor $= \dfrac{16570}{220} = 75$ amps.

Shunt field current $=2$ amps.

Armature current $=75-2=73$ amps.

Torque $=\dfrac{\text{H.P.}\times 33000}{2\pi N}=\dfrac{20\times 33000\times 7}{2\times 22\times 1000}$

$=105$ lb. ft.

Back emf. e $=220-73\times{\cdot}075=220-5{\cdot}47$

$=214{\cdot}53$ volts.

At 500 rpm.

(*a*) The H.P. output of the motor

$$=\frac{\text{Torque}\times 2\pi N}{33000}$$

$$=105\times 2\times\frac{22}{7}\times\frac{500}{33000}$$

$=10$ H.P.

(*b*) We know torque $=KI_a\phi$.

But as the flux and torque remain the same, therefore, the armature current $I_a=73$ amps.

(*c*) The back emf. $=\dfrac{p\phi ZN}{10^8 q 60}$

$=$A constant $\times\phi\times N$.

The flux being constant and the speed being half the former value, the back emf. $=\dfrac{214{\cdot}53}{2}=107{\cdot}26$ volts.

The voltage across the armature

$=e+I_aR_a$

$=107{\cdot}26+5{\cdot}47=112{\cdot}73$

The voltage absorbed by the external resistance

$=220-112{\cdot}73=107{\cdot}27$ volts.

External resistance $=\dfrac{107{\cdot}27}{73}=1{\cdot}47$ ohms.

(*d*) Motor output $=10\times 746=7460$ watts.

Input $=220\times(73+2)=16500$ watts.

Efficiency $=\dfrac{7460\times 100}{16500}=45{\cdot}2\%$. *Ans.*

Example 7. A 4 H.P. 230 V. shunt motor running at 1,400 rpm. has an armature resistance of 1·25 ohms and a shunt field resistance of 460 ohms. Its stray power loss is 125 watts. (*a*) Find the value of the armature current for which the armature efficiency is a maximum and find the value of this maximum efficiency. (*b*) Find the overall maximum efficiency of the machine

(*a*) The armature efficiency will be a maximum when the stray power loss equals the armature copper loss.

Stray power loss $=125$ watts.

If I_a is the armature current when the efficiency of the armature is a maximum

Then
$$I_a^2 R_a = 125$$
$$I_a^2 \times 1{\cdot}25 = 125$$
$$I_a = 10 \text{ amps.}$$

The value of this maximum efficiency

$$= \frac{230\times 10 - 125 - 125}{230\times 10} \times 100$$
$$= 89{\cdot}1\%.$$

(*b*) The overall efficiency is a maximum when

Armature loss $=$ Shunt field loss $+$ Stray power loss

Shunt field loss $= \frac{230}{460}\times 230 = 115$ watts.

$$I_a^2 R_a = 115 + 125 = 240 \text{ watts.}$$
$$I_a^2 = \sqrt{\frac{240}{1{\cdot}25}} = 13{\cdot}86 \text{ amps.}$$

The value of the efficiency

$$= \frac{\text{Input} - \text{losses}}{\text{Input}} \times 100$$
$$\frac{230\times 13{\cdot}86 + 115 - 240 - 240}{230\times 13{\cdot}86 + 115}$$
$$= \frac{3188 + 115 - 480}{3188 + 115} = \frac{2823}{3303} = {\cdot}853$$
$$= 85{\cdot}3\%. \quad \textit{Ans.}$$

CHAPTER XVI

MOTOR STARTERS

16-1. Starting of D.C. Motors.

When a motor is at rest there is no back emf. in its armature and when the machine is started, the back emf. is not established at once. If the machine at rest is directly connected to the supply mains the current flowing through the armature will be $\frac{V}{R_a}$ where V is the supply voltage and R_a the armature resistance. This current is very large and the armature conductors shall be overheated if such a heavy current is allowed to pass through the armature.

When the motor is running at full load, the back emf. e is nearly equal to the applied voltage and the armature current

$$I_a = \frac{V - e}{R_a}$$

The armature is designed to carry this full load current and not the abnormally heavy current that will flow when the back emf. is zero. To avoid this large current at starting a resistance is connected in series with the armature and gradually cut out as the speed and therefore the back emf. increases.

The value of the starting resistance is such that the motor develops sufficient torque at starting. As the motor speeds up, back emf. is generated and the current and torque developed by the machine fall. It is necessary to tap the starting resistance at suitable points and connect them to studs on which the starting arm moves.

For a shunt motor the starting resistance is connected in series with the armature only and not in series with the shunt field. In the case of a series motor the starting resistance is in series with the armature and the field.

16·2. Design of a shunt motor starter.

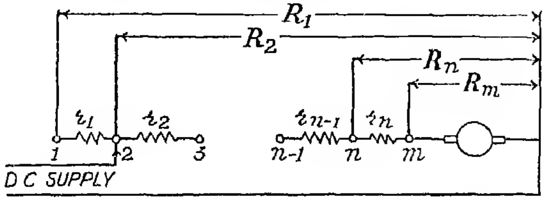


Fig. 115. Shunt Motor Starting Resistance.

There are n resistance elements and $n+1$ studs including the last stud marked m as the motor stud.

Let n=Number of resistance elements.

R_1=Resistance in the armature circuit on the first stud 1 including armature resistance.

R_2=Resistance in the armature circuit on the second step 2 including armature resistance.

R_m=Resistance between armature terminals.

I_1=Maximum value of current.

I_2=Normal full load current
=Minimum value of starting current

V=Line voltage.

e=Back emf.

At stud No. 1 the whole of the resistance R_1 is in the circuit and we have :

$$I_1=\frac{V}{R_1}$$

The motor starts and gains speed developing back emf. e. The current falls to the minimum value I_2.

$$I_2=\frac{V-e}{R_1}$$

or

$$R_1=\frac{V-e}{I_2} \qquad ...(i)$$

Now the starter arm is moved to step No. 2. The resistance now in the armature circuit, including the armature resistance is R_2 and the current rises to its maximum value I_1.

$$I_1=\frac{V-e}{R_2}$$

or
$$R_2=\frac{V-e}{I_1} \qquad \ldots(ii)$$

From equations (*i*) and (*ii*)

$$\frac{I_1}{I_2}=\frac{V-e}{R_2}\times\frac{R_1}{V-e}=\frac{R_1}{R_2}$$

The various resistances in the starter should be such that the ratio $\frac{I_1}{I_2}$ is constant at all steps during starting. In the same way the resistances on successive steps satisfy the conditions :

$$\frac{I_1}{I_2}=\frac{R_1}{R_2}=\frac{R_2}{R_3}=\frac{R_3}{R_4}=\frac{R_{n-1}}{R_n}=\frac{R_n}{R_m}$$

Let
$$\frac{I_1}{I_2}=K$$

Then
$$K^n=\frac{R_1}{R_2}\times\frac{R_2}{R_3}\times\frac{R_3}{R_4}\times\frac{R_{n-1}}{R_n}\times\frac{R_n}{R_m}=\frac{R_1}{R_m}$$

$$n\log K=\log\frac{R_1}{R_m}$$

$$n\log\frac{I_1}{I_2}=\log\frac{R_1}{R_m} \qquad \ldots\text{Eq. (16-1)}$$

From the above

$$R_2=\frac{R_1}{K}$$

$$R_3=\frac{R_2}{K}\text{ etc}$$

The value of I_1 usually equals $1{\cdot}5\ I_2$.

16-3. Design of a series motor starter.

In this case the change in armature current also produces a change in the flux of the main poles.

There are n resistance elements and $n+1$ studs including the last marked m as the motor stud.

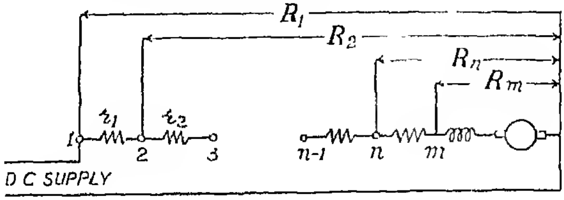


Fig. 116. Series Motor Starting Resistance

Let n=Number of resistance elements.

R_1=Resistance in the motor circuit on the first stud 1 including resistance of the armature and field.

R_2=Resistance in the motor circuit on the second stud marked 2 including armature and field resistances.

I_1=Maximum value of current.

I_2=Minimum value of current.

ϕ_1=Flux corresponding to I_1

ϕ_2=Flux corresponding to I_2

V=Line voltage.

On the contact arm first touching contact 1 the current is maximum and

$$R_1=\frac{V}{I_1}.$$

When the contact arm leaves stud 1 the motor has a back emf. E and the current is a minimum and $E=V-I_2R_1$

When the arm touches stud 2 the current rises to the maximum value I_1 and the flux changes from ϕ_2 to ϕ_1. As the speed cannot change at once the back emf. E changes to E′ due to the change of flux.

$$R_2=\frac{V-E'}{I_1}$$

$$\frac{E'}{E}=\frac{\phi_1}{\phi_2}=y \text{ (say)}$$

$$E'=yE$$

$$R_2=\frac{V-E'}{I_1}=\frac{V}{I_1}-\frac{E'}{I_1}$$

$$=\frac{V}{I_1}-\frac{yE}{I_1}=\frac{V}{I_1}-\frac{y}{I_1}\;(V-I_2\,R_1)$$

$$=\frac{V}{I_1}-\frac{yV}{I_1}+\frac{I_2}{I_1}\,yR_1$$

$$=R_1-yR_1+\frac{y}{K}R_1 \qquad \left[\because \frac{I_1}{I_2}=K\right]$$

$$\therefore\; R_1-R_2 \qquad =y\,R_1-\frac{y}{K}R_1=y\,R_1\left(1-\frac{1}{K}\right)$$

$$\text{or}\quad r_1 \qquad =y\,R_1\left(1-\frac{1}{K}\right) \qquad \text{...Eq. (16-2)}$$

The above equation gives a relation between the first resistance element r_1 of the starter and the total resistance in the motor circuit on the first contact stud. Let us now find the relation between the successive resistance elements r_1, r_2 etc.

When the contact arm leaves contact p the current is a minimum and the motor has a back emf. e volts and

$$e=V-I_2\,R_p.$$

When the arm touches the next stud $(p+1)$ the current rises to the maximum value I_1 and the flux changes from ϕ_2 to ϕ_1. As the speed cannot change at once the back emf. changes from e to e' due to change of flux.

$$e'=V-I_1\,R_{p+1}$$

$$\frac{e}{e'}=\frac{V-I_2\,R_p}{V-I_1\,R_{p+1}}$$

$$\text{or}\qquad \frac{e}{ey}=\frac{V-I_2\,R_p}{V-I_1\,R_{p+1}}$$

$$V-I_1R_{p+1}=y(V-I_2\,R_p)$$

$$=yV-yI_2\,R_p.$$

$$I_1\,R_{p+1}=V-yV+y\,I_2\,R_p.$$

$$R_{p+1}=\frac{V}{I_1}-y\frac{V}{I_1}+y\frac{I_2}{I_1}R_p$$

$$=R_1-y\,R_1+y\times\frac{1}{K}R_p$$

$$=R_1\,(1-y)+\frac{y}{K}-R_p$$

This is a general equation and is true for all values of and

$$R_p = R_1(1-y) + \frac{y}{K} R_{p-1}$$

$$R_p - R_{p+1} = \frac{y}{K}(R_{p-1} - R_p)$$

$$R_p - R_{p+1} = r_p$$

$$R_{p-1} - R_p = r_{p-1}$$

Therefore $$r_p = \frac{y}{K} r_{p-1}$$

or $$r_{p+1} = \frac{y}{K} r_p. \quad \text{..Eq. (16-3)}$$

This shows that the successive resistance elements are in geometrical progression, and the common ratio is $\frac{y}{K}$ (say $=x$).

$$R_1 = r_1 + r_2 + r_3 + \ldots\ldots r_{n-1} + r_n + R_m$$

$$\therefore \quad R_1 = r_1(1 + x + x^2 + \ldots\ldots x^{n-1}) + R_m$$

$$= r_1 \frac{1-x^n}{1-x} + R_m.$$

Hence $$R_1 - R_m = r_1 \cdot \frac{1-x^n}{1-x} \quad \text{...Eq. (16-4)}$$

Rewriting Eq. (16-2)

$$r_1 = y \cdot R_1\left(1 - \frac{1}{K}\right)$$

$$= \frac{y}{K} R_1(K-1)$$

$$= x \cdot R_1(K-1) \quad \text{...Eq. (16-5)}$$

Substituting this value of r_1 in Eq. (16-4)

$$R_1 - R_m = R_1 \cdot x \cdot (K-1) \cdot \frac{1-x^n}{1-x}$$

or $$\frac{R_1 - R_m}{R_1} = x \cdot (K-1) \cdot \frac{1-x^n}{1-x} \quad \text{...Eq. (16-6)}$$

Example 1. A 20 H.P. 250V shunt motor takes full load current of 70 amps. If the maximum starting current is not

to exceed 1·5 times full load current, find the number of steps required in the starting resistance and the resistance of each step. The armature resistance is ·2 ohm.

The total resistance in the circuit at the first contact

$$R_1=\frac{250}{70\times 1\cdot 5}=2\cdot 38 \text{ ohm.}$$

$$R_m=\cdot 2 \text{ ohm.}$$

$$\frac{R_1}{R_m}=\frac{2\cdot 38}{\cdot 2}=11\cdot 9$$

$$n\log\frac{I_1}{I_2}=\text{Log } 11\ 9$$

$$n\log 1\cdot 5=\text{Log } 11\cdot 9$$

$$n\times\cdot 1761=1\ 0755$$

$$\therefore \quad \frac{1\cdot 0755}{\cdot 176}=6$$

$$K=1\cdot 5$$

$$\frac{1}{K}=\frac{1}{1\ 5}=\cdot 667$$

$$R_1=2\ 38 \text{ ohms.}$$

$$R_2=2\cdot 38\times\cdot 667=1\cdot 59 \text{ ohms.}$$

$$R_3=1\cdot 59\times\cdot 667=1\cdot 06 \quad ,,$$

$$R_4=1\cdot 06\times\cdot 667=\cdot 7 \quad ,,$$

$$R_5=\cdot 7\times\cdot 667=\cdot 467 \quad ,,$$

$$R_6=\cdot 467\times 667=\cdot 31 \quad ,,$$

Resistance element $r_1=R_1-R_2=\cdot 79$ ohm.

$$r_2=R_2-R_3=\cdot 53 \quad ,,$$

$$r_3=R_3-R_4=\cdot 36 \quad ,,$$

$$r_4=R_4-R_5=\cdot 233 \quad ,,$$

$$r_5=R_5-R_6=\cdot 157 \quad ,,$$

$$r_6=R_6-R_m=\cdot 11 \quad ,,$$

Example 2. Calculate the number of steps for a shunt motor starter and the resistance of each step for a 6 H.P. 240 volt motor capable of starting under full load. The starting current is not to exceed twice full load current nor to be less

than 1·25 times full load current. Efficiency of the motor is 90% and its armature resistance ·58 ohm.

$$\text{Full load current} = \frac{6\times 746}{90\times 240}\times 100 = 20{\cdot}7 \text{ amps.}$$

Maximum starting current

$$= 2\times 20{\cdot}7 = 41{\cdot}4 \text{ amps.}$$

Resistance in the armature circuit at the first contact

$$R_1 = \frac{240}{41{\cdot}4} = 5{\cdot}8 \text{ ohm.}$$

$$\frac{\text{Max. value of starting current}}{\text{Minimum value of starting current}} = \frac{2\times 20{\cdot}7}{1{\cdot}25\times 20{\cdot}7}$$

$$= \frac{2}{1{\cdot}25}$$

$$n \log \frac{2}{1{\cdot}25} = \log \frac{5{\cdot}8}{{\cdot}58}$$

$$n \times {\cdot}2011 = 1$$

$$n = 5$$

$$\frac{1}{K} = \frac{1{\cdot}25}{2} \quad {\cdot}625$$

$$R_1 = 5{\cdot}8 \text{ ohms.}$$

$$R_2 = 5{\cdot}8\times{\cdot}625 = 3{\cdot}625 \text{ ohms.}$$

$$R_3 = 3{\cdot}625\times{\cdot}625 = 2{\cdot}265 \text{ ohms.}$$

$$R_4 = 2{\cdot}265\times{\cdot}625 = 1{\cdot}415 \quad ,,$$

$$R_5 = 1{\cdot}415\times{\cdot}625 = {\cdot}884 \text{ ohm.}$$

Resistance element $r_1 = R_1 - R_2 = 2{\cdot}175$ ohms.

$r_2 = R_2 - R_3 = 1{\cdot}36$,,

$r_3 = R_3 - R_4 = {\cdot}85$ ohm.

$r_4 = R_4 - R_5 = {\cdot}531$,,

$r_5 = R_5 - R_m = {\cdot}304$,,

Example 3. Calculate the resistances of the steps of a starter suitable for a 450 V, 15 H.P. shunt motor having an efficiency of 85%, the starting torque needed being very low. The armature resistance is 1 ohm and the maximum starting

current must not exceed 1·5 times full load current. There should be 6 resistance elements in the starter.

Output $= 15$ H.P.

Full load current $= \dfrac{15 \times 746}{450 \times 85} = 29 \cdot 2$ amps.

Maximum starting current

$= 1 \cdot 5 \times 29 \cdot 2 = 43 \cdot 8$ amps.

Total resistance in the circuit at the first step

$$= R_1 = \frac{450}{43 \cdot 8} = 10 \cdot 27 \text{ ohms.}$$

$$n \log K = \log \frac{R_1}{R_m}$$

$$6 \log K = \log 10 \cdot 27$$

$$\log K = \frac{1 \cdot 0116}{6} = \cdot 1686$$

$$K = 1 \cdot 474$$

$$\frac{1}{K} = \frac{1}{1 \cdot 474} = \cdot 678$$

$R_1 = 10 \cdot 27$ ohms.

$R_2 = 10 \cdot 27 \times \cdot 678 = 6 \cdot 963$ ohms.

$R_3 = 6 \cdot 963 \times \cdot 678 = 4 \cdot 721$ „

$R_4 = 4 \cdot 721 \times \cdot 678 = 3 \cdot 2$ „

$R_5 = 3 \cdot 2 \times \cdot 678 = 2 \cdot 17$ „

$R_6 = 2 \cdot 17 \times \cdot 678 = 1 \cdot 47$ „

Resistance element $r_1 = R_1 - R_2 = 3 \cdot 307$ ohms.

$r_2 = R_2 - R_3 = 2 \cdot 242$ „

$r_3 = R_3 - R_4 = 1 \cdot 521$ „

$r_4 = R_4 - R_5 = 1 \cdot 03$ „

$r_5 = R_5 - R_6 = \cdot 7$ ohm.

$r_6 = R_6 - R_m = \cdot 47$ „

Example 4. Calculate the resistance of the various steps and the number of steps for a 15 H.P 500 V series motor. Armature and field resistance is 1·25 ohms. Assume that the motor starts on full load with a current variation between

twice and 1·5 times full load value. Motor efficiency is 80%. Assume that the fluxes increases 10% when the current rises from 1·5 to 2 times full load value.

$$\text{Kw. input} \qquad = \frac{15 \times 746}{\cdot 8 \times 1000} = 12{\cdot}4 \text{ Kw.}$$

$$\text{Full load current} \qquad = \frac{12{\cdot}4 \times 1000}{500} = 24{\cdot}8 \text{ amps.}$$

Let R_1 be the total resistance at start

$$R_1 = \frac{500}{24{\cdot}8 \times 2} = 10{\cdot}1 \text{ ohms.}$$

r_1 the first resistance element

$$= yR_1\left(1 - \frac{1}{K}\right)$$

$$y = \frac{\phi_1}{\phi_2} = 1{\cdot}1$$

$$\frac{1}{K} = \frac{I_2}{I_1} = \cdot 75$$

$$r_1 = 1{\cdot}1 \times 10{\cdot}1\left(1 - \frac{3}{4}\right) = 2{\cdot}8 \text{ ohms.}$$

$$r_2 = r_1 \times \frac{y}{K} = 2{\cdot}8 \times 1{\cdot}1 \times \cdot 75$$

$$= 2{\cdot}8 \times \cdot 825 = 2{\cdot}3 \text{ ohms.}$$

$$r_3 = 2{\cdot}28 \times \cdot 825 = 1{\cdot}89 \text{ ohms.}$$

$$r_4 = 1{\cdot}88 \times \cdot 825 = 1{\cdot}55 \text{ ohms.}$$

$$r_m = 1{\cdot}55 \times \cdot 825 = 1{\cdot}27 \text{ ohms.}$$

The total resistance R_1 comes to be

$$r_1 + r_2 + r_3 + r_4 + r_m = 2{\cdot}8 + 2{\cdot}3 + 1{\cdot}89 + 1{\cdot}55 + 1{\cdot}25$$

$$= 9{\cdot}8 \text{ ohms.}$$

This is quite near the value of 10·1 ohms, found at the beginning.

Example 5. A 10 h.p. 220 volt, series motor has the magnetization curve given by

I (amps)	60	48	36	24	12	0
ϕ (megalines)	0·750	0·675	0·562	0·412	0·180	0

Find the resistance of each of the 4 elements. The current limit is 60 amperes. The resistance of the motor is 0·5 ohm. What is the minimum value of current.

$$I_1=60 \text{ amps.}$$

$$\therefore \quad \phi_1=0{\cdot}750 \text{ megalines}$$

$$R_1=\frac{220}{60}=3{\cdot}66 \text{ ohms}$$

$$R_m=0{\cdot}5 \text{ ohms}$$

$$n=4.$$

$$\frac{R_1-R_m}{R_1}=\frac{3{\cdot}66-0{\cdot}5}{3{\cdot}66}=0{\cdot}864$$

Assuming different values of I_2, let us calculate

$$x.(K-1).\frac{(1-x)^n}{1-x}$$

I_2	48	36	24
$\therefore \quad K=\left(\frac{I_1}{I_2}\right)$	1·25	1·67	2·5
$\therefore \quad \Phi_2$	0·675	0·562	0·412
$\therefore \quad y=\frac{\Phi_1}{\Phi_2}$	1·11	1·33	1·82
$\therefore \quad x=\frac{y}{K}$	·89	·796	·728
x^4	0·627	0·403	0·28
$1-x^4$	0·373	0·597	0·72
$\frac{1-x^4}{1-x}$	3·48	2·93	2·65
$\therefore \quad x.(K-1)\frac{1-x^4}{1-x}$	·773	1·59	2·98

Plot a curve between I_2 and $x.(K-1).\frac{1-x^4}{1-x}$.

Reading the value of I_2 for which $x.(K-1).\frac{1-x^4}{1-x}$

$$=\cdot 864,$$

we get $I_2=46$ amps.

$\therefore$ Minimum current $=46$ amps.

$$\therefore \quad K=\frac{60}{46}=1\cdot 3$$

The value of ϕ_2 against $I_2=46$, (by plotting a curve between ϕ and I) is $=\cdot 66$

$$\therefore \quad y=\frac{\phi_1}{\phi_2}=\frac{\cdot 75}{\cdot 66}=1\cdot 137$$

$$\therefore \quad x=\frac{y}{K}=\frac{1\cdot 137}{1\cdot 3}=\cdot 874$$

Substituting in Eq. (16-5)

$$r_1=\cdot 874\times 3\cdot 66\times\cdot 3=\cdot 960\Omega$$
$$r_2=\cdot 874\times\cdot 96=0\cdot 839\Omega$$
$$r_3=\cdot 874\times\cdot 839=0\cdot 732\Omega$$
$$r_4=\cdot 874\times\cdot 732=\cdot 640$$

Total $\quad 3\cdot 171$

R_1-R_m given $=3\cdot 66-\cdot 5$

$=3\cdot 16.$ *Ans.*

CHAPTER XVII

EFFICIENCY TESTS ON D.C. MACHINES

17·1. The efficiency of a D.C. machine can be determined by the following 3 methods :

(*a*) Direct Method.

(*b*) Indirect Method—Swinburn Method.

(*c*) Regenerative Method—Hopkinson's Method.

(*a*) To measure the efficiency by the direct method it is necessary to measure the power input and output of the generator. The efficiency can then be calculated. In the case of a motor the input can be measured by an ammeter and a voltmeter and the output deduced by loading the machine with a brake.

The direct method is seldom used because it is a wasteful method. This is only adopted in laboratories for small machines.

Brake Test on motor. (*Rope brake*).

The common type of brake employed is the rope brake. The load can be regulated by the weight carried by the pan. The actual torque is given by

$$T=(W-w)\times R \text{ lb. ft.} \qquad \text{...Eq. (1·17)}$$

where W is the weight carried by the pan in lbs.

w is the reading of the spring balance in lbs.

R is the effective radius in feet.

$$=\frac{1}{2}\text{(Dia. of pulley+Dia. of rope)}$$

Then Power output

$$=\frac{{}^{\prime}\pi NT}{33000} \qquad \text{...Eq. (17·2)}$$

where N=Rev. per minute

Note. A belt can be used instead of a rope with two spring balances attached on both sides of it. The difference in the readings of the spring balances is then W−w.

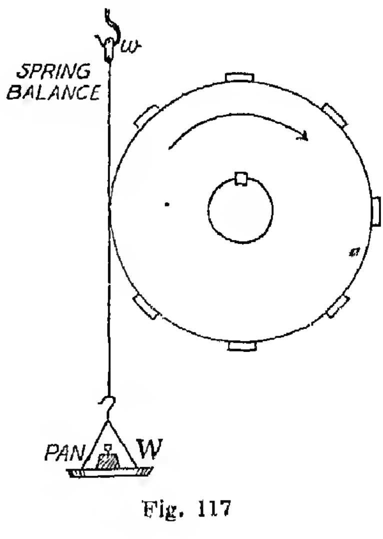


Fig. 117

(*b*) **Indirect method—Swinburn's method.**

In this method the machine is not directly loaded. The copper losses are calculated by measuring the resistances of the various parts. The iron and

friction losses are determined by measuring the input to the machine when it is run as a motor at no load at the rated voltage and speed This method can be used for shunt and compound motors.

Shunt motor efficiency.

Armature current at no load

$$=I_0$$

Iron, friction and windage losses

$$=VI_0$$

(The no load armature copper loss is neglected because it is very small)

R_a= Total resistance of the main motor circuit (Armature, Brush contacts and Interpoles)

R_{sh}=Shunt field resistance

Then for a motor current I

$$I_a = I - \frac{V}{R_{sh}}$$

Armature circuit copper loss

$$=I_a^2R_a$$

$$\text{Efficiency} = \frac{VI - I_a^2R_a - \frac{V^2}{R_{sh}} - VI_0}{VI}$$

...Eq. (17-3)

(*c*) **Regenerative method or Hopkinson's Test.**

In this method two similar direct current machines are needed. One of the machines is run upto speed in the normal manner as a shunt motor. The other machine mechanically coupled to the motor is then excited to such a degree that it gives a voltage equal to the supply voltage feeding the motor.

The generator is put in parallel with the supply system, the excitation being adjusted till the voltmeter V′ reads zero and then switch S i put on.

Under these conditions the generator neither gives nor takes any current from the supply. Any desired load can be put on the generator by increasing its induced emf. This can be done by increasing the generator excitation or by decreasing

the motor excitation or by doing both adjustments. The current generated can then be used to help drive the machine

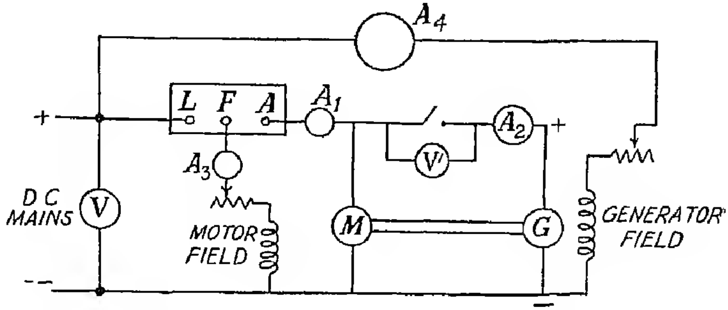


Fig. 118. Hopkinson's Test on D.C. shunt machine

running as a motor and the total current taken from the supply system is, therefore, simply that needed for the losses.

During the test the supply voltage should be kept at its normal value and the speed should be the normal speed of the machines.

At the end of the experiment the resistance of the armature circuits should be taken by ammeter and voltmeter method.

The efficiency can be found as shown below—

I = Current taken by the motor armature from the mains as read by A_1.

I_2 = Generator armature current read by A_2.

I_3 = Motor shunt field current read by A_3.

I_4 = Generator field current read by A_4.

I_1 = Motor armature current $= I + I_2$.

r_1 = Motor armature circuit resistance (hot).

r_2 = Generator armature circuit resistance (hot).

Motor armature circuit copper loss

$$= I_1^2 r_1$$

Generator armature circuit copper loss

$$=I_2^2r_2$$

Power drawn from the supply

$$=VI$$

Total Iron, friction and windage losses for the two machines

$$=VI-(I_1^2r_1+I_2^2r_2)$$

Iron friction and windage losses for each machine

$$=\frac{1}{2}\left[VI-(I_1^2r_1+I_2^2r_2)\right]=W$$

Motor Input $=VI_1+VI_3$

Motor losses $=W+I_1^2r_1+VI_3$

Motor Efficiency $=\dfrac{(VI_1+VI_3)-(W+I_1^2r_1+VI_3)}{VI_1+VI_3}$ ...Eq. (17-4)

Generator output $=VI_2$

Generator losses $=W+I_2^2r_2+VI_4$

Generator efficiency $=\dfrac{VI_2}{VI_2+W+I_2^2r_2+VI_4}$ ...Eq. (17-5)

Example 1. In a brake test on a 5 H.P., 500 volts shunt motor the following readings were obtained :—

Effective dia. of brake drum

$=15''$

$W=32{\cdot}4$ lbs.

$w=6{\cdot}1$ lbs.

Speed $=1600$ r.p.m.

Supply volts $=500$

Armature current $=8{\cdot}5$ amps.

Field current $={\cdot}5$ amp.

Determine the efficiency of the motor.

$$\text{Torque } T=(W-w)R=(32{\cdot}4-6{\cdot}1)\times\frac{7{\cdot}5}{12}$$

$$=16{\cdot}4 \text{ lb. ft.}$$

Power output $= \frac{2\pi NT}{33000} = \frac{2\pi \times 1600 \times 16 \cdot 4}{33000}$

$= 5$ HP.

$= 5 \times 746 = 3730$ watts.

Power Input $= V(I_a + I_{sh})$ watts.

$= 500(8 \cdot 5 + \cdot 5) = 500 \times 9$

$= 4500$ watts.

Efficiency $= \frac{3730}{4500} \times 100 = 83\%$. *Ans.*

Example 2. In a brake test on a shunt motor the following readings were taken.

Supply voltage $= 220$ volts.

Motor Input current $= 17$ amps.

Speed $= 1500$ rpm.

Load on one side of belt

$= 90$ lbs.

Load on the other side $= 30$ lbs.

Effective dia. of pulley $= 6$ inches.

Calculate the torque, BHP and the efficiency of the motor

Torque T $= (W - w) \times R$ lb. ft.

$= (90 - 30) \times \frac{6}{12 \times 2} = 15$ lb. ft.

BHP of the motor $= \frac{2 \times \pi \times 1500 \times 15}{33000}$

$= 4 \cdot 28$ H.P.

Efficiency $= \frac{4 \cdot 28 \times 746}{220 \times 17} = 84 \cdot 8\%$. *Ans.*

Example 3. A. D.C. 230 volt compound machine has the following particulars :—

Armature and Brush contact resistance

$= 2 \cdot 2$ ohms.

Shunt field resistance $= 460$ ohms.

Series field resistance $= \cdot 25$ ohm.

When the machine is run light as a motor at its normal speed and rated voltage, it takes an armature current of ·6

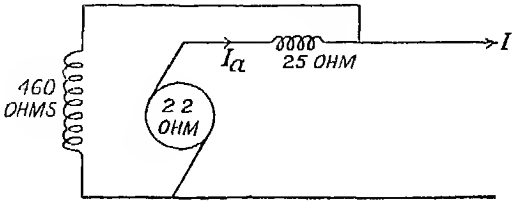


Fig. 119

amp. Find its overall efficiency, connected long shunt compound and used as a generator delivering (*a*) 1·5 Kw. (*b*) ·75 Kw. The resistances given are at the normal working temperature.

(*a*) Current output when power delivered is 1·5 Kw.

$$=\frac{1500}{230}=6\cdot5 \text{ amps.}$$

Shunt field loss $=\frac{230}{460}\times 230=115$ watts.

Armature current $=6\cdot5+\frac{230}{460}=7$ amps.

Armature and series field loss

$$=7\times7\times(2\cdot2+\cdot25)$$

$$=120 \text{ watts.}$$

Iron, friction and windage losses

$$=230\times\cdot6=138 \text{ watts.}$$

Efficiency $=\frac{\text{output}}{\text{output}+\text{losses}}$

$$=\frac{1500}{1500+115+120+138}=\frac{1500}{1873}$$

$$=\cdot8=80\%.$$

(*b*) Current output when power delivered is ·75 Kw.

$$=\frac{750}{230}=3\cdot25 \text{ amps.}$$

Armature current $=3{\cdot}25+{\cdot}5=3{\cdot}75$ amps.

Armature and series field loss

$$=3{\cdot}75^2\times 2{\cdot}45=34{\cdot}4 \text{ watts.}$$

Efficiency $$=\frac{750}{750+115+34{\cdot}4+138}=\frac{750}{1037{\cdot}4}$$
$$={\cdot}723=72{\cdot}3\%.$$

Example 4. Find the overall efficiency of the machine of example 3 connected long shunt compound and used as a motor giving an output (*a*) 4 BHP, (*b*) 2 BHP and (*c*) one BHP.

(*a*) Shunt field loss $=115$ watts.

Iron, friction windage losses

$$=138 \text{ watts.}$$

If I_a is the armature and series field current when the machine gives an output of 4 BHP

$$230\ I_a-2{\cdot}45\ I_a^2-138=4\times 746=2984$$
$$230\ I_a-2{\cdot}45\ I_a^2-3122=0$$
$$-2{\cdot}45\ I_a^2+230\ I_a-3122=0$$

$$I_a=\frac{-230\pm\sqrt{230^2-4\times 2{\cdot}45\times 3122}}{-4{\cdot}9}$$
$$=\frac{-230\pm\sqrt{52900-30595}}{-4{\cdot}9}$$
$$=\frac{-230\pm\sqrt{22305}}{-4{\cdot}9}$$
$$=\frac{-230\pm 159}{-4{\cdot}9}=\frac{81}{4{\cdot}9}=16{\cdot}5 \text{ amp.}$$

Armature loss and series field loss

$$=16{\cdot}5^2\times 2{\cdot}45=667 \text{ watts.}$$

Total losses $=115+138+667=920$ watts.

Efficiency $$=\frac{2984}{2984+920}=\frac{2984}{3904}={\cdot}764$$
$$=76{\cdot}4\%.$$

(*b*) In this case the value of I_a shall be obtained from the equation

$$-2\cdot45\ I_a^2+230\ I_a-138=2\times746=1492$$

$$-2\cdot45\ I_a^2+230\ I_a-1630=0$$

$$I_a=\frac{-230\pm\sqrt{52900-15974}}{-4\cdot9}$$

$$=\frac{-230\pm\sqrt{36926}}{4\cdot9}=\frac{-230\pm192}{4\cdot9}$$

$$=\frac{38}{4\cdot9}=7\cdot76\text{ amps.}$$

Armature and series field loss

$$=7\cdot76^2-2\cdot45=147\text{ watts.}$$

Total losses $=115+138+147=400$

Efficiency $=\dfrac{1492}{1492+400}=\dfrac{1492}{1892}$

$$=\cdot788=78\cdot8\%.$$

(*c*) In this case the value of I_a shall be obtained from the equation

$$-2\cdot45\ I_a^2+230\ I_a-138=746$$

$$-2\cdot45\ I_a^2+230\ I_a-884=0$$

$$I_a=\frac{-230\pm\sqrt{52900-4\times2\cdot45\times884}}{-4\cdot9}$$

$$=\frac{-230\pm\sqrt{52900-8663}}{4\cdot9}$$

$$=\frac{-230\pm\sqrt{44237}}{-4\cdot9}=\frac{-230+210}{4\cdot9}$$

$$=\frac{20}{4\cdot9}=4\cdot08\text{ amps.}$$

Armature and series field loss

$$=4\cdot8^2\times2\cdot45=40\cdot76\text{ watts.}$$

Total losses $=115+138+40\cdot76=293\cdot76$ watts.

Efficiency $=\dfrac{746}{746+293\cdot76}\quad\dfrac{746}{1039\cdot76}$

$$=\cdot718=71\cdot8\%.\quad \textit{Ans.}$$

Example 5. In the Hopkinson's test on two similar shunt machines the following test results were obtained :

Current taken by the motor armature from the mains
$=40$ amps.

Generator armature current$=260$ amps.

Generator field current$=5$ amps.

Motor field current $=4{\cdot}4$ amps.

Armature resistance of each machine$=0{\cdot}05$ ohm.

Line volts. $=460$ volts.

Find the efficiency of each machine.

I =Current taken by motor armature from the line
$=40$ amps.

I_2=Generator armature current$=260$ amps.

I_1=Motor armature current$=260+40=300$ amps.

I_3=Motor field current$=4{\cdot}4$ amps.

I_4=Generator field current$=5$ amps.

$r_1=r_2=0{\cdot}05$ ohm. =Armature resistance of each.

Iron, friction and windage loss per machine

$$=W=\frac{1}{2}\left[VI-(I_1^2r_1+I_2^2r_2)\right]$$

$$=\frac{1}{2}\left[460\times40-(300^2\times{\cdot}05+260^2\times{\cdot}05)\right]$$

$$=\frac{1}{2}\left[18400-(4500+3380)\right]$$

$$=5260 \text{ watts.}$$

Motor input
$$=VI_1+VI_3$$
$$=(460\times300)+(460\times4{\cdot}4)$$
$$=138000+2040=140040 \text{ watts.}$$

Motor losses
$$=W+I_1^2r_1+VI_3$$
$$=5260+4500+2040=11800 \text{ watts.}$$

Motor efficiency
$$=\frac{140040-11800}{140040}=\frac{128240}{140040}$$
$$={\cdot}916=91{\cdot}6\%.$$

Generator output $=VI_2=460\times 260=119600$

Generator losses $=W+I_2^2r_2+VI_4$

$=5260+3380+460\times 5$

$=10940$ watts.

Generator efficiency $=\dfrac{119600}{119600+10940}=\dfrac{119600}{130540}$

$=\cdot 916=91\cdot 6\%$. *Ans.*

Example 6. The Hopkinson's test on two similar shunt machines gave for full load the following results :—

Line voltage $=200$ volts,

Line current excluding field currents $=10$ amps.

Motor armature current $=70$ amps.

Motor field current one amp. and Generator field current $=1\cdot 2$ amps.

Armature resistance of each machine is $0\cdot 2$ ohm. Calculate the efficiency of each machine.

$I=$ Current taken by motor armature from line $=10$ amps.

$I_1=$ Motor armature current $=70$ amps.

$I_2=$ Generator armature current $=I_1-I=60$ amps.

$I_3=$ Motor field current $=1$ amp.

$I_4=$ Generator field current $=1\cdot 2$ amps.

$r_1=r_2=$ Armature resistance of each machine $=0\cdot 2$ ohm.

$V=$ line voltage $=200$ volts.

Iron, friction and windage loss per machine

$$=W=\frac{1}{2}\left[VI-(I_1^2r_1+I_2^2r_2)\right]$$

$$=\frac{1}{2}\left[200\times 10-(70^2\times\cdot 2+60^2\times\cdot 2)\right]$$

$=150$ watts.

Motor input $=VI_1+VI_3=200\times 70+200\times 1$

$=14200$ watts.

Motor losses $=W+I_1^2r_1+VI_3$

$=150+70^2\times\cdot 2+200\times 1$

$=1330$ watts.

Motor efficiency $=\dfrac{14200-1330}{14200}=\dfrac{12870}{14200}$

$=\cdot 906=90\cdot 6\%$.

Generator output $=VI_2=200\times 60=12000$ watts.

Generator losses $=W+I_2{}^2r_2+VI_4$

$=150+60^2\times\cdot 2+200\times 1\cdot 2$

$=150+720+240=1110$ watts.

Generator output $=\dfrac{12000}{12000+1110}=\dfrac{12000}{13110}=\cdot 915$

$=91\cdot 5\%$. *Ans.*

CHAPTER XVIII

PARALLEL OPERATION OF D.C. GENERATORS

18-1. When supplying a large load it is usual to instal a number of small units rather than one large unit capable of dealing with the maximum load. The former arrangement has the advantage that the number of units running can be varied with the load so as to maintain individual generators at approximately full load or near about the load that gives highest efficiency. It has the additional advantage that the power station is not seriously crippled owing to the breakdown of one generating unit.

18-2. Shunt generators in parallel.

Shunt machines when run in parallel are stable in their operation. They divide the load among themselves according to their capacities and load characteristics. For small changes of speeds the electrical interaction of the machines tends to equalise the speeds and loads of the machines.

Suppose two machines M_1 and M_2 equally loaded are operating in parallel. If the prime mover of machine M_1 slightly slows down in speed then its emf. will fall and M_2 shall be taking a larger share of the load. Because a smaller load is being taken by M_1 the voltage drop in the armature of M_1 will decrease and so also its armature reaction. Both these will tend to raise its emf. and terminal p.d. On the other hand, the greater load on machine M_2 tends to decrease its emf. Thus the two machines tend to equalise.

Switch Boards.

The machines when connected in parallel are connected through switches, fuses, and other protective devices mounted on panels as shown in fig. 120.

18-3. Compound generators in parallel.

In this case equaliser bars are used to connect the series fields in parallel. The effect of the equaliser is to divide the

load properly between the generators. Without the equaliser the machines are unstable.

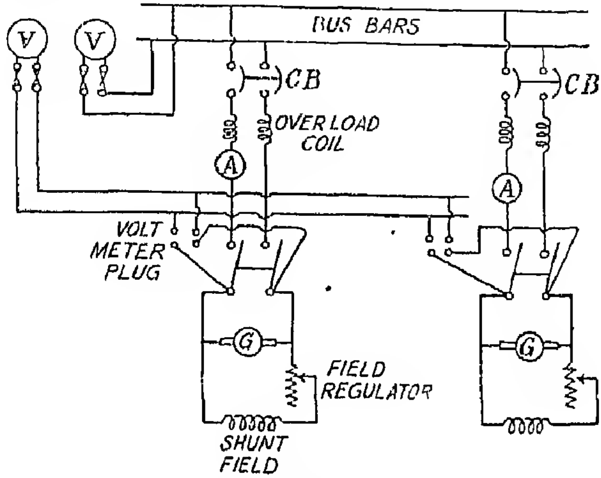


Fig. 120
Switch board diagram for operating two shunt generators in parallel.

If there is no equaliser connection and the speed of one generator slightly goes up, then its voltage increases and it takes a greater share of the load. This greater load strengthens its series field further and further raises its emf. and the load. Thus this generator tends to take up the entire load and to operate the other machine as a motor.

If the equaliser connection is provided the increased speed in one machine increases its voltage and current output. But the increased current output instead of flowing through the series field of one generator divides itself at the brush, a part flowing through the series winding of each generator. Thus the voltage of each generator is equally increased and the equilibrium is not disturbed.

To start and connect a compound generator in parallel.

The field resistance should be in the maximum position when the machine is started. When the machine has come to normal speed, cut out the field resistance till the voltage is normal or equal to or a trifle above that of the bus bars. Now put on the equaliser switch first and then the two other switches. If a 3 pole switch is used, all the three poles are, of course, connected at the same time.

Two or more identical compound machines with equaliser bars distribute the loads equally among themselves at all loads.

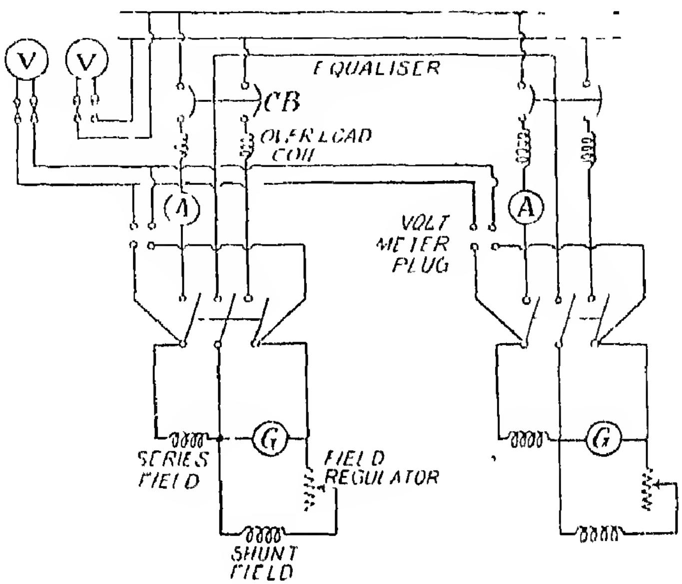


Fig. 121.
Switch board diagram for operating two compound generators in parallel.

There is difficulty if the characteristics are different. Compound generators of different types can work satisfactorily in parallel, if both the machines have the same degree of compounding or over compounding when running separately, otherwise the machine with the higher degree of over compounding will take the greater load.

Example 1. Two separately excited generators are operating in parallel and supply equal currents at a terminal voltage of 120 volts to a load resistance whose value is ·4 ohm. Generator A has an armature resistance of ·04 ohm. and generator B ·05 ohm. The machines are driven by separate prime movers at a constant speed and the field excitations are constant.

(*a*) What will be the terminal voltage and the current supplied by each machine if the load resistance is changed to one ohm.,

(b) What will happen if the load is taken off the system.

Fig. 122

Current supplied to the load

$$=\frac{120}{\cdot 4}=300 \text{ amps.}$$

Current supplied by each machine

$$=150 \text{ amps.}$$

Emf of A $=120+150\times\cdot 04=126\text{V.}$

Emf of B $=120+150\times\cdot 05=127\cdot 5\text{V.}$

(a) If the load resistance is one ohm then

Current supplied by machine A

$$=I_1=\frac{126-V}{\cdot 04},$$

Current supplied by $B=I_2=\frac{127\cdot 5-V}{\cdot 05}$, where V is the common voltage

$$I_1+I_2=\frac{V}{R}, \text{ R is the load resistance}$$

or $$\frac{126-V}{\cdot 04}+\frac{127\cdot 5-V}{\cdot 05}=\frac{V}{1}=V.$$

$$3150-25V+2550-20V=V.$$

$$46V=5700$$

$$V=123\cdot 9V.$$

$$I_1=\frac{126-123\cdot 9}{\cdot 04}=\frac{2\cdot 1}{\cdot 04}=52\cdot 5 \text{ amps.}$$

$$I_2=\frac{127\cdot 5-123\cdot 9}{\cdot 05}=\frac{3\cdot 6}{\cdot 05}=72 \text{ amps.}$$

$$I_1+I_2=\frac{V}{1}=123\cdot 9 \text{ amps.}$$

(*b*) When the load is taken off.

Resultant emf in the armature circuit

$$= 127{\cdot}5 - 126$$
$$= 1{\cdot}5\text{V}.$$

Resistance in the armature circuit

$$= {\cdot}04 + {\cdot}05$$
$$= {\cdot}09 \text{ ohm}.$$

Circulating current in the armatures

$$= \frac{1{\cdot}5}{{\cdot}09} = 16{\cdot}6 \text{ amps}.$$

Example 2. Two D.C. shunt generators are connected in parallel and supply jointly a total load of 1000 amps. The

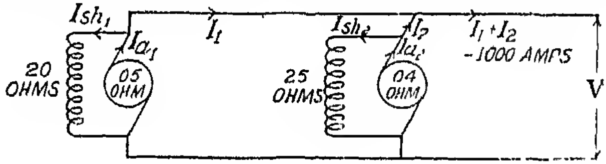


Fig. 123

machines have armature resistance of ·05 and ·04 ohm, field resistances of 20 and 25 ohms and give emfs. of 440 and 420 volts respectively. Determine (*a*) the current and power output of each machine (*b*) At what total load current will the terminal voltage be 410 volts.

(*a*) Let I_1 and I_2 be the currents sup plied to the external circuit by the two machines respectively and let V be the common terminal voltage.

$$I_1 + I_2 = 1000 \text{ amps}.$$

$$I_{sh1} = \frac{V}{20};\quad I_{sh2} = \frac{V}{25}$$

$$I_{a1} = \frac{440 - V}{{\cdot}05},\quad I_{a2} = \frac{420 - V}{{\cdot}04}$$

$$I_1 = I_{a1} - I_{sh1} = \frac{440 - V}{{\cdot}05} - \frac{V}{20}$$

$$I_2 = I_{a2} - I_{sh2} = \frac{420 - V}{{\cdot}04} - \frac{V}{25}$$

$$\therefore \qquad \left(\frac{440-V}{\cdot05}-\frac{V}{20}\right)+\left(\frac{420-V}{\cdot04}-\frac{V}{25}\right)=1000$$

$$88\ 0-20V-\cdot05V+10500-\cdot25V-\cdot04V=1000$$

$$V=405\cdot85 \text{ volts.}$$

$$I_1=\frac{440-405\cdot85}{\cdot05}-\frac{405\cdot85}{20}=662\ 7 \text{ amps.}$$

$$I_2=\frac{420-405\ 85}{\cdot04}-\frac{405\ 85}{25}=337\cdot3 \quad ,,$$

$$\text{Output of Machine } 1=\frac{405\cdot85\times662\ 7}{1000}=269 \text{ Kw}$$

$$\text{Output of Machine } 2=\frac{405\cdot85\times337\ 3}{1000}=137 \text{ Kw}$$

(*b*) When terminal voltage is 410 volts

$$I_{a1}=\frac{440-410}{.05}=\frac{30}{\cdot05}=600 \text{ amps.}$$

$$I_{a2}=\frac{420-410}{\cdot04}=\frac{10}{\cdot04}=250 \text{ amps.}$$

$$I_{sh1}=\frac{410}{20}=20\cdot5 \text{ amps.}$$

$$I_{sh2}=\frac{410}{25}=16\cdot4 \text{ amps.}$$

$$I_1=I_{a1}-I_{sh1}=600-20\ 5=579\cdot5 \text{ amps.}$$

$$I_2=I_{a2}-I_{sh2}=250-16\cdot4=233\cdot6 \text{ amps.}$$

$$\text{Total load current} \quad =I_1+I_2=579\cdot5+233\cdot6=813\cdot1 \text{ amps.}$$

Example 3. Two D. C. generators are operating in parallel. Their characteristics may be represented by straight lines joining the points 235V, 41 amp., 260V, 0 amp. in the case of one machine and 220V, 50 amp., 280V, 0 amp. in the other. Determine the current of each machine and the voltage of the combination for total load currents of (*a*) zero, (*b*) 30 amps, (*c*) 50 amps.

1st Method. (*Calculation*)

Machine 1 : Voltage drops by 25 volts when current increases from zero to 41 amps. or 6 volt per amp.

Machine 2 : Voltage drops by 60 volts when current rises from 0 to 50 amps. or 1·2 volts per amp.

(*a*) Load current = Zero ; if V is the common terminal voltage and I_1 and I_2 the respective currents supplied by machines 1 and 2, we have

$$260 - \cdot 6\, I_1 = 280 - 1\cdot 2\, I_2$$

or $$1\cdot 2\, I_2 - \cdot 6\, I_1 = 20$$

Multiply by $\frac{5}{6}$

$$I_2 - \cdot 5\, I_1 = \frac{50}{3}$$

Also $$I_2 + I_1 = 0$$

$$1\cdot 5\, I_1 = -\frac{50}{3}$$

$$I_1 = -\frac{100}{9} = -11\cdot 1 \text{ amps.}$$

This shows that machine 1 receives a current of 11·1 amps. and machine 2 delivers a current of 11·1 amps.

The terminal p.d. $V = 280 - 1\cdot 2 \times 11\cdot 1$
$= 266\cdot 7$ volts

(*b*) Load current is 30 amps.

Here $$I_2 + I_1 = 30$$

$$I_2 - \cdot 5\, I_1 = \frac{50}{3}$$

$$1\cdot 5\, I_1 = 30 - \frac{50}{3} = \frac{40}{3}$$

$$I_1 = 8\cdot 9 \text{ amps.}$$

$$I_2 = 30 - 8\cdot 9 = 21\cdot 1 \text{ amps.}$$

$$V = 260 - \cdot 6 \times 8\cdot 9 = 254\cdot 6 \text{ volts.}$$

(*c*) Load current 50 amps.

$$I_2 + I_1 = 50$$

$$I_2 - \cdot 5\, I_1 = \frac{50}{3}$$

$$1{\cdot}5\,I_1 = 50 - \frac{50}{3} = \frac{100}{3}$$

$$I_1 = \frac{200}{9} = 22{\cdot}2 \text{ amps.}$$

$$I_2 = 50 - 22{\cdot}2 = 27{\cdot}8 \text{ amps.}$$

$$V = 260 - {\cdot}6 \times 22{\cdot}2 = 246{\cdot}7 \text{ volts.}$$

Method 2 (*Graphical*).

Curve I shows the external characteristic of machine 1 and curve II that of machine 2. To draw the combined external characteristic of the two machines draw a line O′C parallel to OX, cutting curves I and II at *a* and *b* respectively. Make O′C equal to O′*a*+O′*b*. Then C is a point on the composite characteristic. Similarly mark another point on the composite characteristic, join the points and produce on both sides.

(*a*) **Load current zero.**

The common terminal voltage as shown by point A on curve III where it cuts the axis OY is 267 volts. Draw a line parallel to OX from point A. It cuts curve I and curve II at −11 and 11 amps. respectively. Thus when the load current is zero, machine 2 gives out 11 amps. and machine 1 receives 11 amps.

$$I_1 = -11 \text{ amps.}$$

$$I_2 = +11 \text{ amps.}$$

(*b*) **Load current 30 amps.**

Point B on the composite characteristic shows this point. If a line parallel to OX is drawn from this point it cuts curves I and II at 9 and 21 amps. respectively.

Common terminal voltage = 225 volts.

$$I_1 = 9 \text{ amps.}$$

$$I_2 = 21 \text{ amps.}$$

(*c*) **Load current 50 amps.**

Point D on curve III represents this point and a line through this point parallel to OX gives

Common terminal voltage = 246 volts,

$$I_1 = 22 \text{ amps.}$$

$$I_2 = 28 \text{ amps.}$$

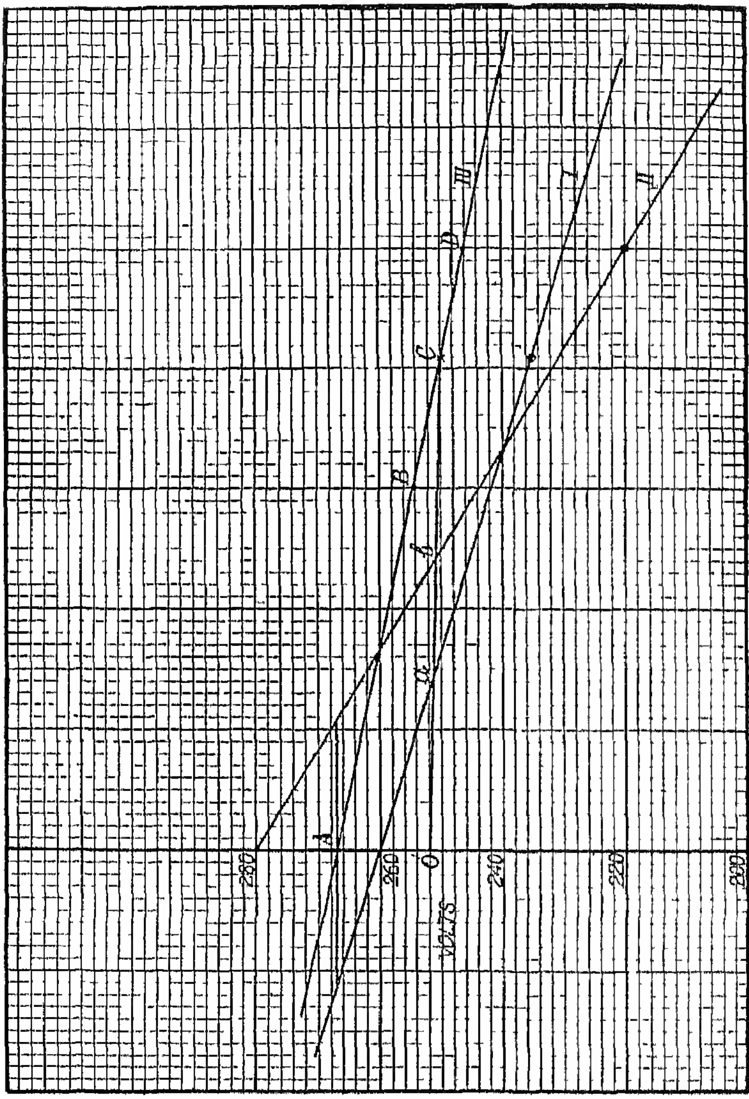

CURRENT
10
20
30
40
50
60
VOLTS
280
260
240
220
200
O
A
B
C
D
I
II
III

Fig. 124

Example 4. Two shunt generators with emfs 120 and 115 volts, armature resistances ·05 and ·04 ohm and shunt field resistances 20 and 25 ohms respectively are running in parallel and supply a total load of 25 Kw. What portion of this load does each machine supply.

Let I_{a1}, I_{a2} be the respective armature currents of the two machines, I_{sh1} and I_{sh2} the respective field currents and V the common terminal voltage

$$I_{a1}=\frac{120-V}{\cdot05}$$

$$I_{a2}=\frac{115-V}{\cdot04}$$

$$I_{sh1}=\frac{V}{20},$$

$$I_{sh2}=\frac{V}{25}$$

Current supplied to the load by the two machines

$$=\left(\frac{120-V}{\cdot05}-\frac{V}{20}+\frac{115-V}{\cdot04}-\frac{V}{\cdot05}\right)\text{ amps.}$$

$$V\left(\frac{120-V}{\cdot05}-\frac{V}{20}+\frac{115-V}{\cdot04}-\frac{V}{25}\right)=25\times1000$$

$$V[2400-20V-\cdot05V+2875-25V-\cdot04V]=25000$$

$$V[5275-45\cdot09\,V]=25000$$

$$-45\cdot09V^2+5275V-25000=0$$

$$V=\frac{-5275\pm\sqrt{5275^2-4\times45\cdot09\times25000}}{90\cdot18}$$

$$V=112V$$

Load current supplied by 1st machine

$$=\frac{120-112}{\cdot05}-\frac{112}{20}$$

$$=160-5\cdot6=154\cdot4\text{ amps.}$$

Load current supplied by the second machine

$$=\frac{115-112}{\cdot04}-\frac{112}{25}$$

$$=75-4\cdot48=70\cdot52\text{ amps.}$$

Example 5. Two generators running in parallel supply a total load current of 150 amps. The terminal p.d. of one machine falls from 250 volts. on no load to 234 volts when its current output is 100 amps. The terminal p.d. of the other machine falls from 250V. on no load to 230 volts when its current output is 80 amps. The external characteristics are rectilinear. Find the current output of each machine and the terminal voltage.

Method 1. (*By Calculation*)

Machine 1 :

Voltage falls 16 volts for a current of 100 amps.

or ·16 volt per amp.

Machine 2 :

Voltage falls 20 volts for current output of 80 amps.

or ·25 volt per amp.

$$\therefore \quad 250-{\cdot}16I_1=250-{\cdot}25I_2=V$$

$$\cdot 16I_1-{\cdot}25I_2=0$$

$$\cdot 64I_1-I_2=0$$

But

$$I_1+I_2=150$$

$$1{\cdot}64I_1=150$$

$$I_1=91{\cdot}4 \text{ amps.}$$

$$I_2=150-91{\cdot}4=58{\cdot}6 \text{ amps.}$$

$$V=250-{\cdot}25I_2$$

$$=250-{\cdot}25\times 58{\cdot}6=250-14{\cdot}65$$

$$=235{\cdot}35 \text{ volts.}$$

Method 2. (*Graphical*)

The external characteristics of machines 1 and 2 are drawn as shown by I and II. Then the composite external characteristic III is drawn. The current of 150 amps. is marked as point C on curve III and the terminal voltage 235·5V read off. The currents supplied by individual machines for this voltage are read off from characteristics I and II as 58·5 and 91·5 amps.

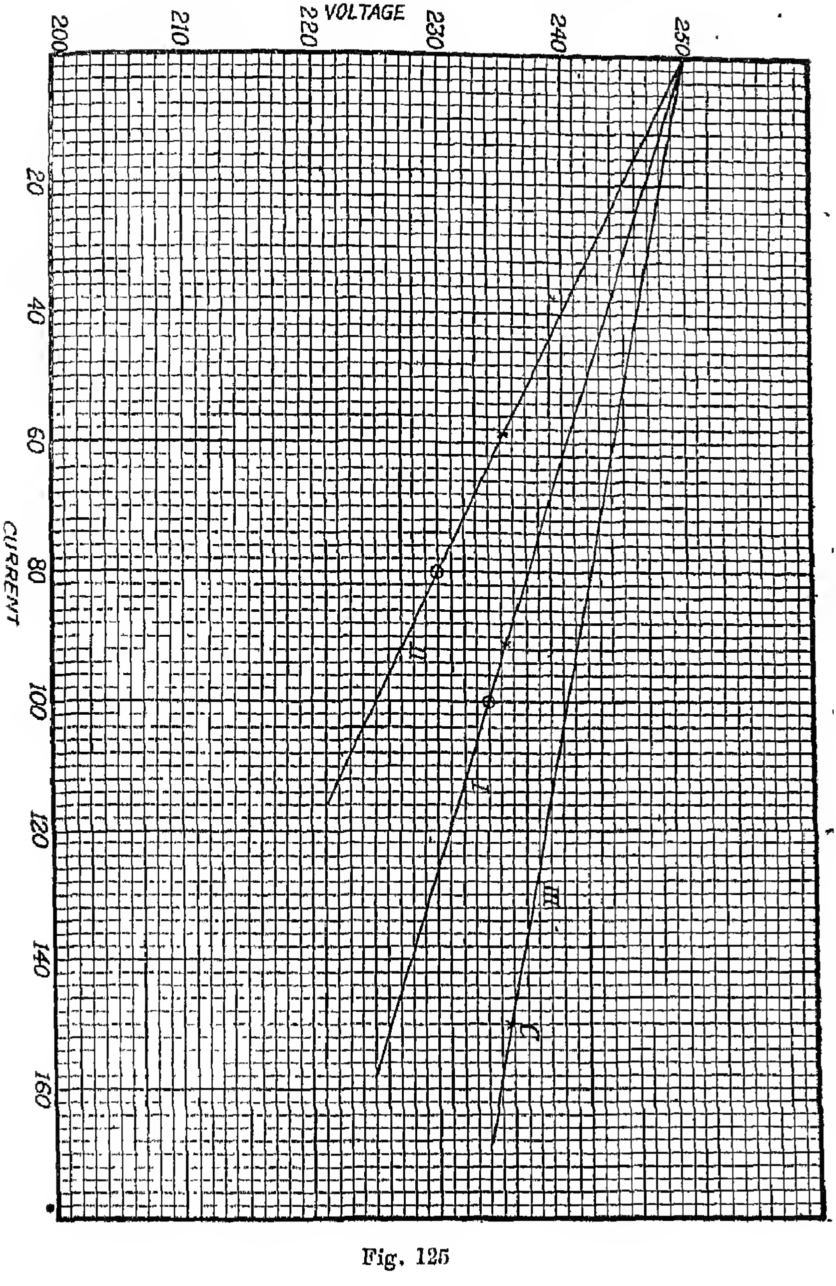
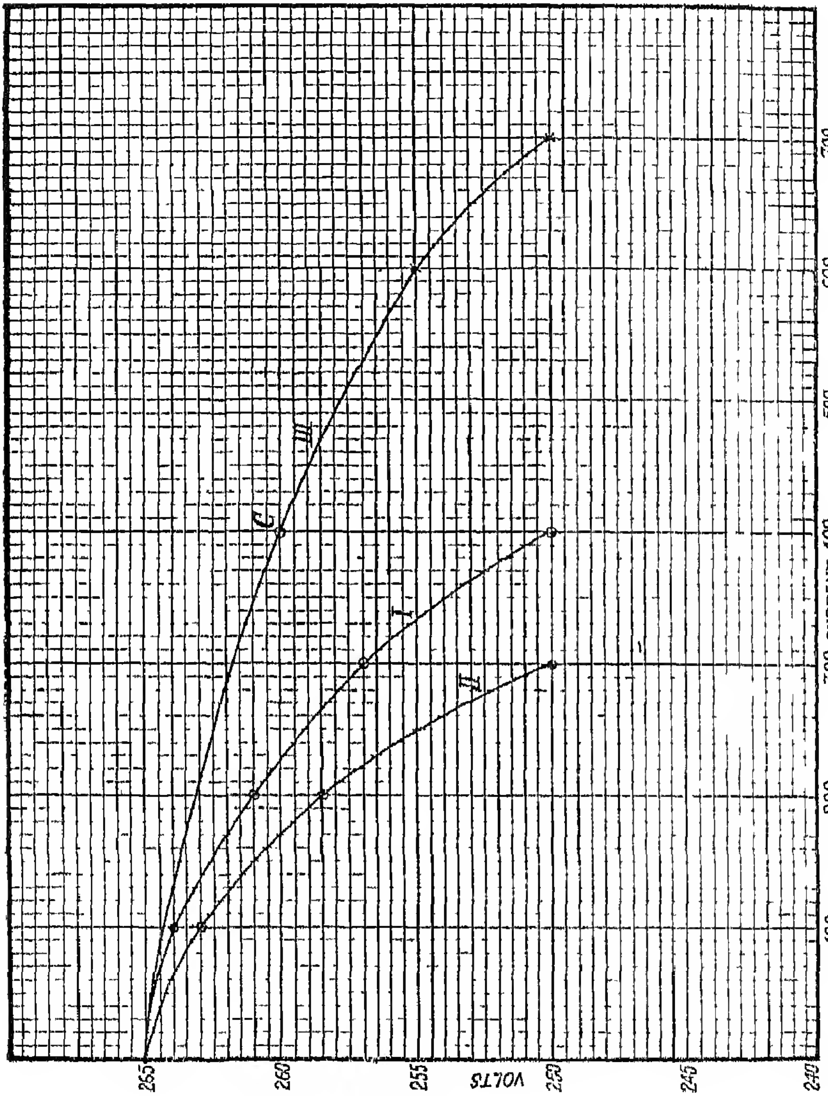

VOLTAGE
200
210
220
230
240
250
CURRENT
20
40
60
80
100
120
140
160
II
I
III
C

Fig. 125



265
260
255
VOLTS
250
245
240
100
200
300 CURRENT 400
500
600
700
I
II
III
C

Fig. 126

Example 6. Two D C. shunt generators of 100 and 75 Kw capacity operate in parallel. Their external characteristics are as follows :

100 Kw machine	I	0	100	200	300	400
	V	265	264	261	257	250
75 Kw machine	I	0	100	200	300	
	V	265	263	258.5	250	

(*a*) They share a load of 400 amps. What is the current supplied by each machine and what is the common bus-bar voltage.

(*b*) The load of 400 amps. is required to be shifted to the 100 Kw machine and the 75 Kw machine is to be taken off the bus-bars. Find the terminal voltage of the generator when this is done by (*i*) increasing the excitation of the bigger machine (*ii*) decreasing the excitation of the smaller machine.

This can only be solved graphically. Plot the external characteristic curves I and II for the 100 Kw and 75 Kw machines. Then plot the composite external curve III for the machines.

(*a*) Point C on curve III reads 400 A. and 260 V. The common terminal voltage is 260 V.

The 260 V. line cuts curves I and II at 230 and 170 amps.

Currents supplied by the machines are 230 and 170 amps. respectively

(*b*) (*i*) The terminal voltage of 110 Kw generator on load should be raised to 265 V and it will carry the total load of 400 amps.

(*ii*) The terminal voltage of the 75 Kw machine should be reduced to 250 V. on no load so that the total load of 400 A is shifted to the 100 Kw machine.

Example 7. A battery whose emf is 110 volts and internal resistance ·12 ohm is being charged by a shunt generator the external characteristic of which is :

Current	0	20	40	60	80	100
Terminal p.d.	125	124	122.5	120	116	111

(*a*) Find the charging current that is being taken by the battery.

(*b*) What resistance should be connected in series with the battery circuit to reduce the charging current to 40 amps

(*a*) Plot the dynamo external characteristic as shown in Fig. 127.

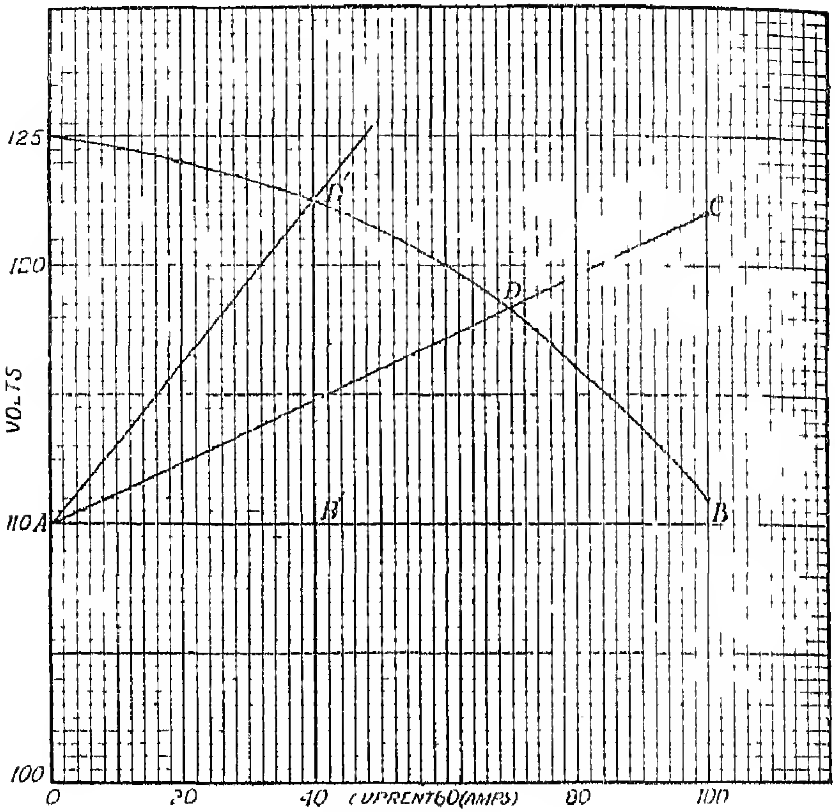


Fig 127

A current of say 100 amps to flow through the battery requires a voltage of $110+100\times\cdot12=122$ volts.

Draw the battery charge line AC connecting point A (0 A, 110 V) and C(100 A, 122 V). The line AC cuts the curve at point D which gives the charging current as 70 amps.

(*b*) For a charging current of 40 amps the battery charge line should cut the dynamo characteristic at point D′ representing 40 amps The total resistance in the battery circuit now

$$=\frac{D'B'}{40}=\frac{12\cdot5}{40}=\cdot3125 \text{ ohm.}$$

Battery resistance $=\cdot 12$ ohm.

Resistance to be connected in series

$$=\cdot 3125-\cdot 12=\cdot 1925 \text{ ohm.} \quad \textit{Ans}$$

Example 8. The terminal voltage of a generator falls uniformly from 250 volts to 240 volts when the current output is 80 amps It is connected to a load in parallel with a battery having a constant emf of 245 volts and an internal resistance of 0 1 ohm Find how a load of 120 amps would be shared.

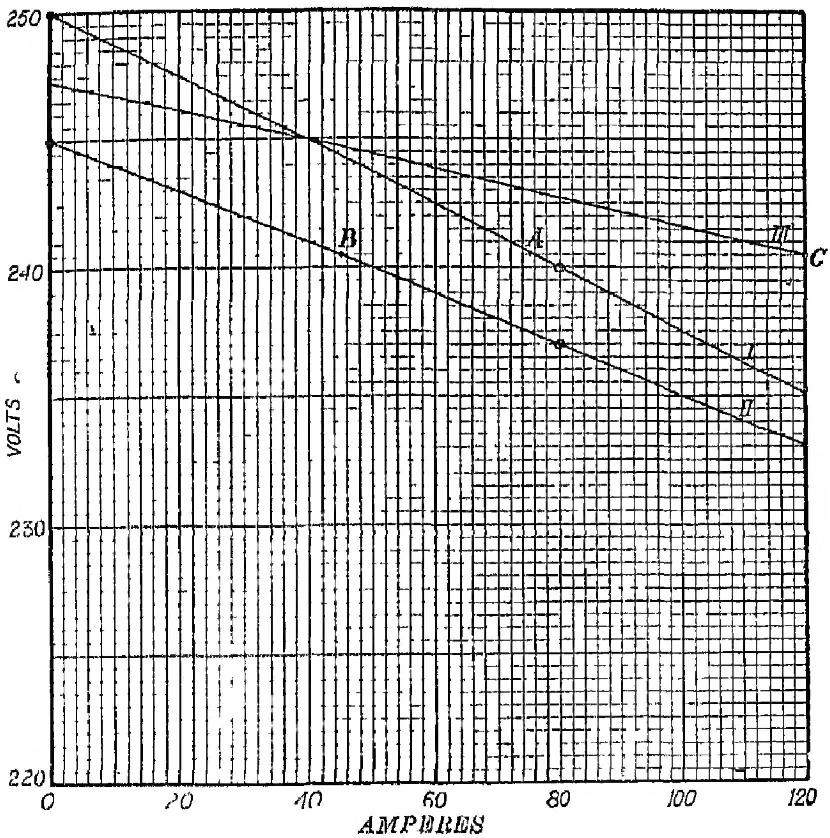


Fig. 128.

Draw the external characteristic I of the generator by connecting points 0 amp. 250 V. and 80 amps. 240 V. and producing p.d. across the battery when its output is say 80 amps.

$$=245-80\times\cdot 1=237 \text{ volts.}$$

The battery external characteristic II is thus obtained by joining the point (0 amp. 245 V.) with the point (80 A. 237 V.)

The composite external characteristic III is then drawn by adding the currents delivered by both the dynamo and the battery at different voltages.

The 120 amps line cuts the composite external characteristic at 240·5 volts at point C. This 240·5 volts line cuts characteristics I and II at points A and B representing 75 amps and 45 amps as the currents delivered by the generator and battery respectively.

Example 9. Four compound generators operating in parallel supply a load of 1600 amps. The resistances of armatures and series field windings of the various machines are given in the diagram. The excitations are so adjusted that the emfs of the machines are as indicated. Find the current output of each machine, the current in each section of the equaliser bar and the bus-bar voltage. Neglect shunt field currents.

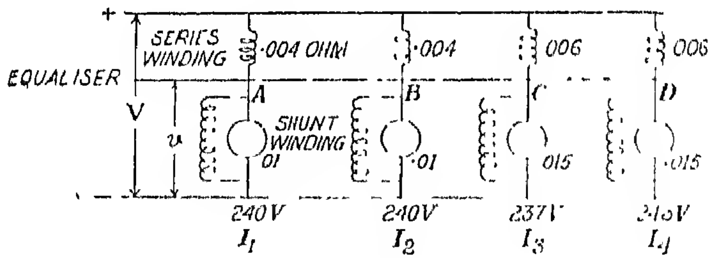


Fig. 129.

Let V be the bus-bar voltage and

v=Voltage between equaliser bus-bar and negative.

I_1, I_2, I_3 and I_4=Current output of the machines.

Then $I_1+I_2+I_3+I_4=1600$

$$\frac{240-v}{\cdot 01}+\frac{240-v}{\cdot 01}+\frac{237-v}{\cdot 015}+\frac{243-v}{\cdot 015}=1600$$

Divide by 100 throughout

$$240-v+240-v+158-\frac{v}{1\cdot5}+162-\frac{v}{1\cdot5}=16$$

$$3\frac{1}{3}v=784$$

$$v=235\cdot2 \text{ volts.}$$

$$I_1=\frac{240-235\cdot2}{\cdot01}=480 \text{ amps.}$$

$$I_2=\frac{240-235\cdot2}{\cdot01}=480 \text{ amps.}$$

$$I_3=\frac{237-235\cdot2}{\cdot015}=120 \text{ amps.}$$

$$I_4=\frac{243-235\cdot2}{0.5}=520 \text{ amps.}$$

The total current of 1600 amps divides amongst the series windings in the ratio

$$\frac{1}{\cdot004};\ \frac{1}{\cdot004};\ \frac{1}{\cdot006};\ \frac{1}{\cdot006} \text{ or } 3:3:2:2.$$

Currents in the series windings are

480, 480, 320, 320 amps.

Current in equaliser bus-bar.

Section AB $=0$

,, BC $=0$

,, CD $=520-320=200$ amps. from D to C.

Bus-bar voltage V $=v+$drop in any series winding

$=235\cdot2+480\times\cdot004$

$=237\cdot12$ volts. *Ans.*

Example 10. Two over compounded generators rated at 100 and 200 Kw are excited such that each has an open circuit voltage of 220 volts. The machines are then connected in parallel and together they supply a total load of 200 Kw. The full load voltage regulations are 4% and 3% respectively. Assuming a straight line relation between the load and the terminal voltage find the load taken by each and the terminal voltage.

Note. Voltage Regulation at full load

$$=\frac{\text{Full load voltage}-\text{No load voltage}}{\text{No load voltage}}$$

Let W_1 be the load on 100 Kw machine

W_2 the load on 200 Kw machine and V the common terminal voltage.

Then $W_1 \times \frac{\cdot04 \times 220}{100}$ = Increase in the p.d. of machine no. 1

and $W_2 \times \frac{\cdot03 \times 220}{200}$ = Increase in the p.d. of machine no. 2

As V is the common terminal voltage

$$\therefore \quad W_1 \times \frac{8\cdot8}{100} = W_2 \times \frac{6\cdot6}{200}$$

$$\text{or} \quad W_1 = W_2 \times \frac{6\cdot6}{200} \times \frac{100}{8\cdot8} = \frac{3}{8} W_2$$

$$\text{and} \quad W_1 + W_2 = 200$$

$$\frac{3}{8} W_2 + W_2 = 200$$

$$W_2 = 200 \times \frac{8}{11} = 145\cdot4 \text{ Kw.}$$

$$W_1 = 200 - 145\cdot4 = 54\cdot6 \text{ Kw.}$$

$$V - 220 = \frac{W_2 \times 6\cdot6}{200} = \frac{145\cdot4 \times 6\cdot6}{200} = 4\cdot8 \text{ V.}$$

$$V = 220 + 4\cdot8 = 224\cdot8 \text{ volts.} \quad \textit{Ans.}$$

CHAPTER XIX

D.C. TRANSMISSION AND DISTRIBUTION

19-1. By transmitting and distributing system or network is meant the cables, connections etc. by which the electric power is conveyed from the switch board in the central station to the places where it is to be used. These cost nearly as much as the machinery in the central station. The conditions to be satisfied by such a system are—

(*a*) The maximum current flowing must not over-heat the conductors and their insulation.

(*b*) The p.d. between the mains at all points where power is used must be maintained within certain limits.

(*c*) The power wasted in it must not exceed a moderate percentage of the power transmitted.

(*d*) The cost of network must not be very high.

19-2. Voltage drop and Efficiency of transmission.

Voltage drop or "Drop" means the difference between the voltage at the bus bars and that between the mains at some other place. In the simplest case of transmission by two wires each of resistance R ohms going to a single place of utilisation the drop is 2IR where I amps is the current transmitted. The drop at any smaller distance shall be less in proportion as the value of R shall be proportionately less.

Efficiency of transmission

$$=\frac{\text{Power given out at the receiving end}}{\text{Power input from the supply station}}$$

If E′ and E be the voltages at the receiving end and the supply station respectively,

$$\text{Efficiency} \quad =\frac{E'I}{EI}=\frac{E-2IR}{E}=1-\frac{2IR}{E}$$

$$=\left(100-\frac{2IR}{E}\times 100\right) \text{ per cent.}$$

or Percentage efficiency$=$100$-$Percentage drop.

Voltage and distance of transmission.

If the distance of transmission is increased and no other change made, the drop is increased in the ratio of the distance. The efficiency of transmission is decreased. If the distance is doubled then the efficiency would be reduced from 95% to 90% or from 90% to 80%.

If it is desired to maintain the efficiency at its former value then two methods are available :

(*a*) Increase the cross-section of the conductors in proportion to the increase of distance thus keeping their resistance at its former value The voltage drop for a given current then remains the same and the efficiency remains unaltered. In this method the amount of copper used for the conductors increases as the square of the distance.

(*b*) Increase the voltage in proportion to the distance. With the same current and cross-section of the cables the voltage drop 2IR increases as the distance. The percentage drop $\frac{2IR}{E} \times 100$ remains unaltered because both the numerator and denominator increase in the same proportion. The efficiency, therefore, remains as before.

19-3. Voltage and weight of conductor.

When the power transmitted, the distance and power loss are fixed the weight of conductors varies inversely as the square of the voltage.

Let P = Power to be transmitted

E_1 = Voltage

I_1 = Current

$2R_1$ = Resistance of the two conductors

$$I_1 = \frac{P}{E_1}$$

Power loss $P_1 = 2I_1^2R_1$

Now assume that the voltage is increased to E_2, the power, distance and power lost remaining the same.

Current $$I_2=\frac{P}{E_2}$$

Power loss $$P_2=2I_2^2R_2$$

$$2I_1^2R_1=2I_2^2R_2$$

$$\therefore \frac{R_1}{R_2}=\left(\frac{I_2}{I_1}\right)^2=\left(\frac{P}{E_2}\right)^2\times\left(\frac{E_1}{P}\right)^2=\frac{E_1^2}{E_2^2}$$

This shows that the conductor resistance varies directly as square of the voltage. But the volume or the weight of copper in the conductor of a given length varies inversely as the resistance. Let weight of copper in the two cases be W_1 and W_2. Then

$$\frac{W_1}{W_2}=\frac{E_2^2}{E_1^2}$$

Therefore the conductor weight varies inversely as the square of the voltage when power, distance and loss are fixed. If the voltage of a system is doubled the weight of the conductors needed is $\left(\frac{1}{2}\right)^2$ or $\frac{1}{4}$ of the former value.

19-4. Two wire and three wire systems of distribution.

The voltage used for distribution is settled mainly by lighting considerations. In the United Kingdom 230 V. is the standard and in the U.S.A. 110 V. is usual. For lighting by incandescent lamps higher voltage necessitates more fragile filaments.

In the three wire system the distribution voltage is double that applied to the lamps. It thus combines to some extent the advantages of high pressure distribution and of low pressure lamps. There are three mains, the positive, the negative and the neutral, sometimes called the middle wire. The positive conductor is maintained at a certain pressure above the neutral wire and the negative at the same pressure below the neutral. Thus the voltage between the outers (positive and negative) is twice that between either of them and the middle wire. Lamps

are connected between either outer and the neutral, while the motors can be connected in the same way or directly across the positive and negative.

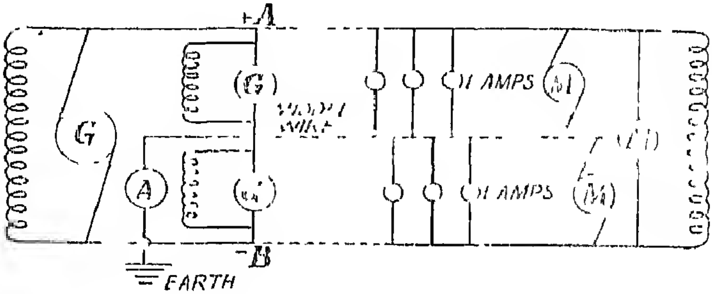


Fig. 130. The three wire system.

G are generators, M are motors.

Balanced and unbalanced loads.

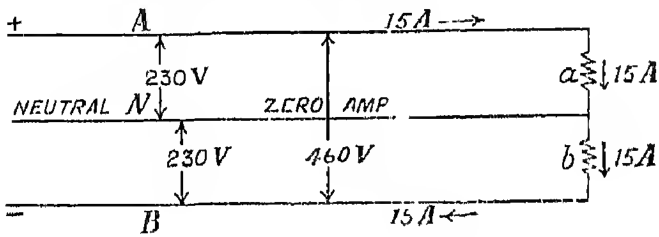


Fig. 131. Three wire balanced system

The sides of the system are said to be balanced when the total load on each side of the system is the same. Figure 131 shows 15 amps. taken by load '*a*' connected between the positive and the neutral and the same current taken by load '*b*' connected on the negative side. Under these conditions the current in the neutral conductor is zero.

But if there is a difference between the two loads, they are called unbalanced and the current in the middle wire is the difference between those in the outers *i.e.*, it is equal to the out of balance current.

Even with balanced loads there will usually be a current in the distributing part of the middle conductor, and this current may be in opposite directions in different parts

of the wire. This can be seen from figure 133. The figures and the arrows indicate the values and directions of the currents.

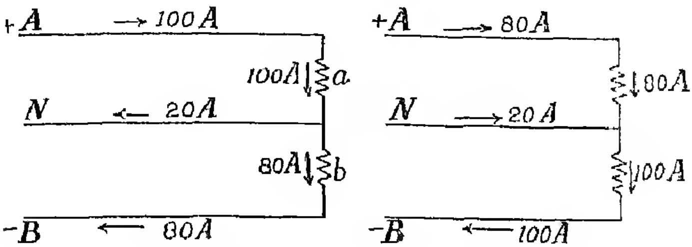


Fig. 132. Three wire unbalanced system.

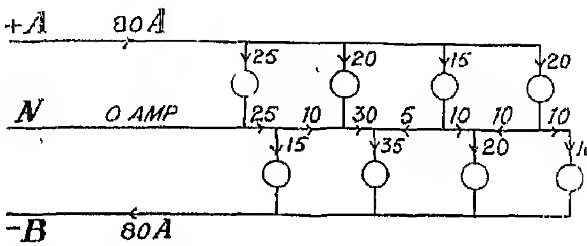


Fig. 133. Currents in three wire distributors.

Saving of copper in three wire system.

We have already shown in this chapter that the conductor weight varies inversely as the square of the voltage when power, distance and loss of power in transmission are fixed. In the three wire system the voltage between the two outers is double the voltage used in the two wire system. The weight of copper of the two outers therefore in the three wire system is $\frac{1}{4}$ that in the two wire system.

The middle wire is frequently of half the cross-section of the outers. Then if the weight of copper in the two wire system is denoted by 100, the following are the weights of copper in the various mains :

In two wire system

	+ ve main	50
	— ve ,,	50
Total		100

In the three wire system

+ ve main	12·5
− ve ,,	12·5
Middle wire	6·25
Total	31·25

This shows that the weight of copper needed in the three wire system is only $31\frac{1}{4}$ % or $\frac{5}{16}$ of that needed in a two wire system for the same purpose.

In the above calculations it is assumed that the carrying capacity of the conductors is not exceeded in the three wire system. This may happen as the current in the outer conductors is halved while the cross-section is made one quarter.

If the current density in the two systems is made equal, the two outers of the three wire system shall need 50 percent copper and the middle wire $12\frac{1}{2}$ % making a total of $62\frac{1}{2}$ %. The losses would then, with balanced loads, be half those in the two wire system and the efficiency of transmission correspondingly higher.

19·5. Series Booster.

A series generator is sometimes used as booster on D.C. feeder when the voltage drop on a feeder becomes excessive.

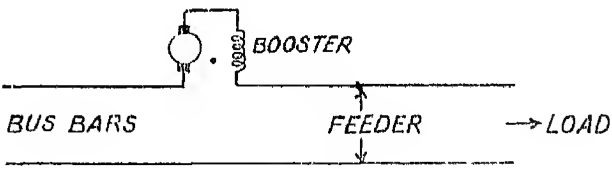


Fig. 134

This machine is a series generator operating on the straight portion of the saturation curve, the terminal voltage being proportional to the current. The voltage drop in the feeder is also proportional to the current. If the booster is connected in series with the feeder (*See* figure 134) and adjusted carefully, its terminal p.d. may be made always equal to the drop in the feeder and the voltage at the load end may be maintained constant.

Two wire distributor fed at one end only.

Example 1. A D.C. distribution cable 350 ft. long has two cores each of 0·1 sq. inch cross-section and is fed at one end at a constant voltage of 250 volts A consumer 'A' whose full load current is 40 amps. is connected to the other end of the distribution cable by a service cable 30 ft. long having two cores each of ·06 sq. in. cross-section. Another consumer B whose full load current is 60 amps. is connected to the distribution cable at a point 150 ft. from connection A by a service cable 50 ft. long with two cores each of ·06 sq. inch. Find the p.d. at each consumer's terminals when both are taking full load. Take the resistivity of copper as ·7 microhms per inch cube.

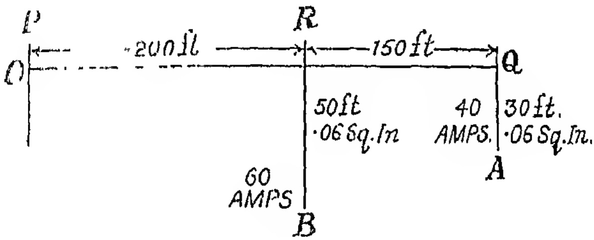


Fig. 135

Current flowing through PR

$$=100 \text{ amps.}$$

Total resistance of cable from P to R (both lead and return)

$$=2\times 200\times 12\times \frac{\cdot 7}{10^6}\times\frac{1}{\cdot 1}$$

$$=4800\times\frac{7}{10^6}\text{ ohm.}$$

Drop. of voltage along PR

$$=4800\times\frac{7}{10^6}\times 100$$

$$=\frac{336}{100}=3\cdot 36\text{ volts.}$$

Resistance of service cable from R to B

$$=2\times 50\times 12\times\frac{\cdot 7}{10^6}\times\frac{1}{\cdot 06}$$

$$=1200\times\frac{\cdot 7}{10^6}\times\frac{100}{6}=\cdot 014\text{ ohm.}$$

Voltage drop in this service cable

$$= \cdot 014 \times 60 = \cdot 84 \text{ volt.}$$

Total voltage drop from the feeding point P to B

$$= 3 \cdot 36 + \cdot 84 = 4 \cdot 2 \text{ volts.}$$

Terminal p.d. at B $= 250 - 4 \cdot 2 = 245 \cdot 8$ volts.

Again resistance of distribution cable from R to Q

$$= 2 \times 150 \times 12 \times \frac{\cdot 7}{10^6} \times \frac{1}{\cdot 1}$$

$$= 3600 \times \frac{7}{10^6} \text{ ohm.}$$

Voltage drop along RQ

$$= 3600 \times \frac{7}{10^6} \times 40$$

$$= 1 \cdot 008 \text{ volts.}$$

Resistance of service cable QA

$$= 2 \times 30 \times 12 \times \frac{\cdot 7}{10^6} \times \frac{1}{\cdot 06}$$

$$= 720 \times \frac{\cdot 7}{10^6} \times \frac{100}{6} = \frac{8 \cdot 4}{1000}$$

Voltage drop in $QA = \frac{8 \cdot 4}{1000} \times 40 = \cdot 336$ volt.

Voltage drop from feeding point P to A

$$= 3 \cdot 36 + 1 \cdot 008 + \cdot 336 = 4 \cdot 704 \text{ volts.}$$

Terminal p.d. at A $= 250 - 4 \cdot 7 = 245 \cdot 3$ volts.

Two wire distributor with uniformly distributed load fed at one end.

Example 2. A 2-core distributor PQ 200 yards long is fed at the end P and supplies a uniformly distributed load of 0·5 amp per yard. The maximum permissible voltage drop is 5 volts. (*a*) Calculate the necessary cross section of the distributor. (*b*) If a concentrated load of 20 amps is added at a point C 50 yards from P, how will the section of the distributor be modified.

Sp. Resistance of copper is ·7 microhm per inch cube.

Taking a general case let $PQ=l$ yards

Current supplied by the feeder per yard length of feeder

$$=i \text{ amp.}$$

Resistance per yard of feeder$=r$ ohm.

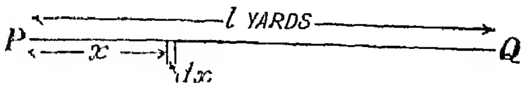


Fig. 136

At point N, x yards from the feeding point P, the current flowing in the feeder $=i\ (l-x)$ amps.

The resistance of the small section dx

$$=r\ dx$$

Voltage drop in this small section

$$=i\ (l-x)\ r\ dx$$

i.e., $$dv=i\ (l-x)\ r\ dx$$

∴ Total voltage drop in length x

$$=ir\left(lx-\frac{x^2}{2}\right)\text{volts.}$$

Total voltage drop in length l is obtained by putting

$$x=l$$

Total voltage drop in length l

$$V=ir\left(l^2-\frac{l^2}{2}\right)$$

$$=\frac{irl^2}{2}=\frac{il\times rl}{2}=\frac{I\times R}{2}$$

where R=Total resistance of the feeder

I=Total current fed at point P

Substituting the values

$$5 = \frac{\cdot 5 \times 200 \times R}{2}$$

or

$$R = \frac{10}{100} = \cdot 1 \text{ ohm.}$$

$$R = \rho \frac{l}{a} = \cdot 7 \times 10^{-6} \times \frac{200 \times 36 \times 2}{a}$$

$$= \frac{100 \cdot 8}{a \times 10^4}$$

$$a = \frac{100 \cdot 8}{10^4 \times \cdot 1} = \frac{100 \cdot 8}{1000} = \cdot 1008 \text{ sq. in.}$$

(b)

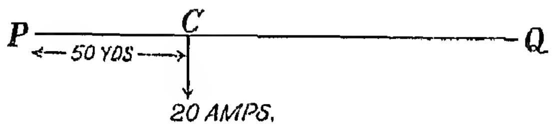


Fig. 137

In this case the drop in voltage is due to the 20 amps. load at C and due to the distributed load along the cables. If a is the cross-sectional area we have

$$\left(50 \times 2 \times 36 \times \frac{\cdot 7}{10^6} \times \frac{1}{a}\right) \times 20$$

$$+ \left(\frac{1}{a} \times \frac{1}{2} \times \cdot 5 \times 200 \times 2 \times 200 \times 36 \times \frac{\cdot 7}{10^6}\right)$$

$$= 5 \text{ volts.}$$

or

$$\frac{504}{10^4 \times a} + \frac{504}{10^3 \times a} = 5$$

$$\frac{504}{10^3 \times a}\left(\frac{1}{10} + 1\right) = 5$$

$$\frac{1 \cdot 1 \times 504}{1000 \times 5} = a = \cdot 111 \text{ sq. in.}$$

Two wire distributor fed at both ends at different voltage.

Example 3. A distributor is fed from both ends. At feeding point A the voltage is maintained at 235 V and at B at

236 volts. The total length of the feeder is 200 yards and loads are tapped off as under.

20 amps. at 50 yards from A
40 amps. at 75 yards from A
25 amps. at 100 yards from A
30 amps. at 150 yards from A

The resistance per 1000 yards of one conductor is ·4 ohm. Calculate the current in the various sections of the feeder, the minimum voltage and the point at which it occurs.

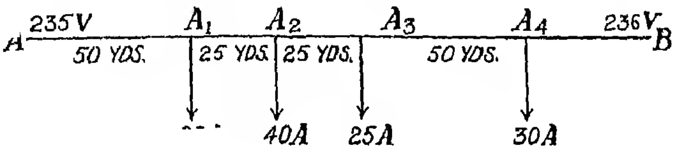


Fig. 138

Total resistance of section AA_1 (lead and return)

$$= \frac{\cdot 4}{1000} \times 50 \times 2 = \cdot 04 \text{ ohm.}$$

Total resistance of section A_1A_2 $= \frac{\cdot 4 \times 25 \times 2}{1000} = \cdot 02$ ohm.

" " " A_2A_3 $= \cdot 02$ ohm.

" " " A_3A_4 $= \cdot 04$ "

" " " A_4B $= \cdot 04$ "

Consider the current flowing from point A to be $= I_A$ amp.

$\therefore$ Voltage drop in section AA_1 $= I_A \times \cdot 04$

Similarly voltage drop in A_1A_2 $= (I_A - 20) \times \cdot 02$

" " A_2A_3 $= (I_A - (0) \times \cdot 02$

" " " A_3A_4 $= (I_A - 85) \times \cdot 04$

" " " A_4B $= (I_A - 115) \times \cdot 04$

Adding all the voltage drops we have

$$I_A \times \cdot 04 + (I_A - 20) \times \cdot 02 + (I_A - 60) \times \cdot 02 + (I_A - 85) \times \cdot 04 + (I_A - 115) \times \cdot 04 = \cdot 16\, I_A - 9 \cdot 6$$

Point B has a voltage one volt above point A.

$$\therefore \qquad \cdot 16\ I_A - 9\cdot 6 = -1$$

$$I_A = \frac{8\cdot 6}{\cdot 16} = 53\cdot 75\ \text{A}$$

Total current $= 20+40+25+30 = 115$ A

Current flowing from B

$= 115 - 53\cdot 75 = 61\cdot 25$ A.

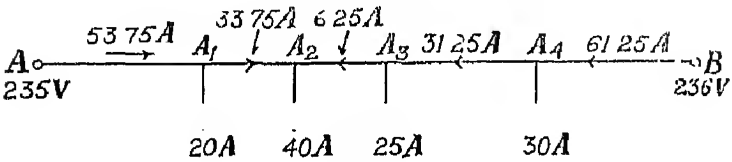


Fig. 130

Current flows from both points A and B to the point A_2. Therefore A_2 is the point of lowest potential.

Taking voltage drop from point A

Drop in AA_1	$= 53\cdot 75 \times \cdot 04 = 2\cdot 15$ V
,, ,, A_1A_2	$= 33\cdot 75 \times \cdot 02 = \ \cdot 675$ V
Total drop	$= \quad 2\cdot 825$ volts

Voltage at point A_2 $= 235 - 2\cdot 825$

$= 232\cdot 175$

Taking voltage drops from point B

Drop in B A_4	$= 61\cdot 25 \times \cdot 04 = 2\cdot 45$ V
,, ,, A_4A_3	$= 31\cdot 25 \times \cdot 04 = 1\cdot 25$,,
,, ,, A_3A_2	$= 6\cdot 25 \times \cdot 02 = \ \cdot 125$,,
Total drop	$= \quad 3\cdot 825$ volts

Voltage at point A_2 $= 236 - 3\cdot 825 = 232\cdot 175$ V.

Ring distributor fed at one point.

Example 4. A 400 yards ring distributor has loads as shown in the figure ; the distances being in yards. The resistance of each conductor is ·2 ohm per 1000 yards The distributor is fed at the point A at 250 volts. Find the voltage at the load points B, C and D.

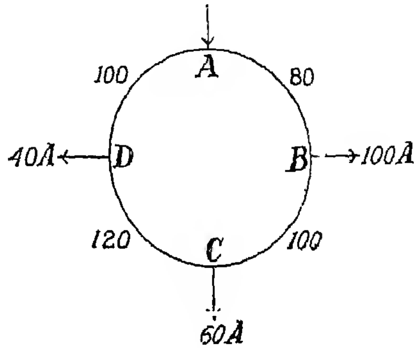


Fig. 140

Resistance of section AB (both conductors)

$$= \frac{2 \times 80 \times \cdot 2}{1000}$$

$$= \cdot 032 \text{ ohm}$$

Resistance of section BC

$$= \frac{2 \times 100 \times \cdot 2}{1000}$$

$$= \cdot 04 \text{ ohm.}$$

Resistance of section

$$CD = \frac{240 \times \cdot 2}{1000} = \cdot 048 \text{ ohm.}$$

Resistance of section $DA = \cdot 04$ ohm.

Let I be the current flowing from A to B, then

$$\cdot 032\ I + (I - 100) \times \cdot 04 + (I - 160) \times \cdot 048 + (I - 200) \times \cdot 04 = 0$$

$$\cdot 032\ I + \cdot 04\ I + \cdot 048\ I + \cdot 04\ I - 19 \cdot 68 = 0$$

$$\cdot 16\ I = 19 \cdot 68$$

$$I = 123 \text{ amps.}$$

Drop in section AB	$123 \times \cdot 032 = 3 \cdot 93$ V.
Voltage at B	$= 250 - 3 \cdot 93 = 246 \cdot 07$ V.
Drop in section BC	$= 23 \times \cdot 04 = \cdot 92$ V.
Voltage at C	$246 \cdot 07 - \cdot 92 = 245 \cdot 15$ V.
Current from A to D	$=$ Total current $- 123 = 77$ amps.
Drop in section AD	$= 77 \times \cdot 04 = 3 \cdot 08$ V.
Voltage at D	$= 250 - 3 \cdot 08 = 246 \cdot 92$ V.
Current from D to C	$= 37$ amps.

Ring distributor fed at two points.

Example 5. A ring main loaded as shown is fed at two points A and B at 250 volts. Find the current in each section of the cable and voltage at each load point. The resistance at each section (both conductors) is given in ohms. Find also the total current supplied at A and B.

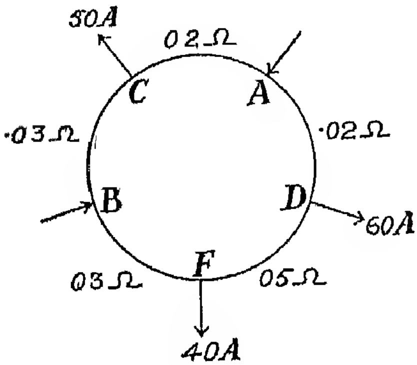


Fig. 141

Let current flowing from A to C $=I_1$

Then $\cdot 02\ I_1+\cdot 03(I_1-50)=0$

$\cdot 02\ I_1+ 03\ I_1=1\cdot 5$

$I_1=30$ amps.

Let current flowing from A to D $=I_2$

Then $\cdot 02\ I_1+ 05(I_2-60)$
$+\cdot 03(I_2-100)=0$

$\cdot 02\ I_2+\cdot 05\ I_2+\cdot 03\ I_2\ \ 6$
$I_2=60$ amps.

Voltage drop in section AD
$=60\times\cdot 02=1\cdot 2$ V.

Voltage at D $=250-1\cdot 2-\ 248\cdot 8$ V.

No current flows in section DF as 60 amps. flowing from A to D goes to the load at D.

$\therefore$ Voltage at point F $=$ Voltage at D
$=248\cdot 8$ V.

Current from B to C $=50-30=20$ amps.

Current from B to F $=40$ amps. for load F

Voltage drop in section AC
$=30\times\cdot 02=\cdot 6$ V.

Voltage at C $=250-\cdot 6\ \ 249\cdot 4$ V.

Current supplied from point A
$=60+30=90$ amps

Current supplied from point B
$=20+40=60$ amps.

Example 6. In the direct current network shown, the feeding point is at A. The resistance of each section (both conductors) is given in ohms. Find the value and direction of current in each section.

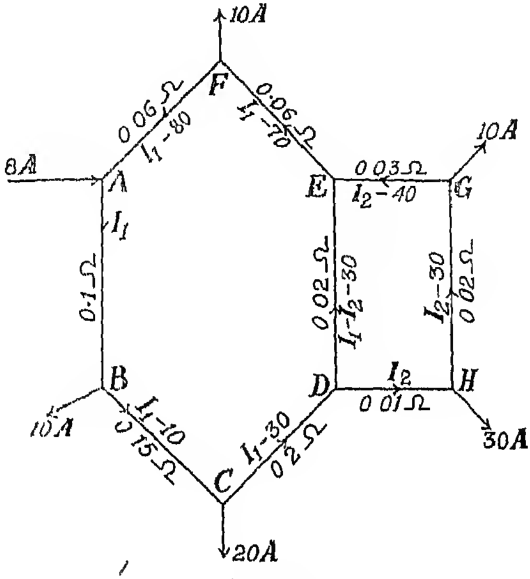


Fig. 142

Suppose a current of I_1 amps. flows from point A to B and a current of I_2 amps. flows from D to H, then the current in each section will be as shown in the figure.

Going round the mesh ABCDHGEF anti-clockwise we have

$$\cdot 1\, I_1 + \cdot 15(I_1-10) + \cdot 2(I_1-30) + \cdot 01 I_2 + \cdot 02(I_2-30) + \cdot 03(I_2-40) + \cdot 06(I_1-70) + \cdot 06(I_1-80) = 0$$

or $\cdot 1\, I_1 + \cdot 15\, I_1 + \cdot 2\, I_1 + \cdot 06\, I_1 + \cdot 06\, I_1 + \cdot 01\, I_2 + \cdot 02\, I_2 + \cdot 03\, I_2 - 1\cdot 5 - 6 - \cdot 6 - 1\cdot 2 - 4\cdot 2 - 4\cdot 8 = 0$

or $\cdot 57\, I_1 + \cdot 06\, I_2 = 18\cdot 3$

Now going round mesh DHGE anticlockwise we have

$$\cdot 01\, I_2 + \cdot 02(I_2-30) + \cdot 03(I_2-40) - \cdot 02(I_1-I_2-30) = 0$$

or $\cdot 01\, I_2 + \cdot 02\, I_2 + \cdot 03\, I_2 - \cdot 02\, I_1 + \cdot 02\, I_2 - \cdot 6 - 1\cdot 2 + \cdot 6 = 0$

or $\quad \cdot 02\ I_1 - \cdot 08\ I_2 = -1\cdot 2 \qquad \ldots(1)$

$\cdot 57\ I_1 + \cdot 06\ I_2 = 18\cdot 3 \qquad \ldots(2)$

Multiply equation (1) by $\frac{3}{4}$

$\cdot 015\ I_1 - \cdot 06\ I_2 = -\cdot 9 \qquad \ldots(3)$

Add equations (2) and (3)

$\cdot 585\ I_1 = 17\cdot 4$

$I_1 = 29\ 7$ amps.

Substitute value of I_1 in equation (1)

$\cdot 02 \times 29\cdot 7 - \cdot 08\ I_2 = -1\cdot 2$

$\cdot 594 - \cdot 08\ I_2 = -1\cdot 2$

$\cdot 08\ I_2 = 1\cdot 794$

$I_2 = 22\cdot 42$ amps.

Currents in various sections

A to B $= I_1 = 29\cdot 7$ amps.

B to C $= I_1 - 10 = 19\cdot 7$ amps.

C to D $= I_1 - 30 = 29\cdot 7 - 30 = -\cdot 3$ amp.

or from D to C $= \cdot 3$ amp.

A to F $= 80 - I_1 = 80 - 29\cdot 7 = 50\ 3$ amps.

F to E $= 70 - I_1 = 40\cdot 3$ amps.

E to G $= 40 - I_2 = 40 - 22\cdot 42 = 17\cdot 58$ amps.

D to H $= I_2 = 22\cdot 42$ amps.

G to H $= 30 - I_2 = 30 - 22\cdot 42 = 7\cdot 58$ amps.

E to D $= I_2 + 30 - I_1$

$= 22\cdot 42 + 30 - 29\cdot 7 = 22\cdot 72$ amps.

Example 7. State Kelvin's Law for the economical section of a feeder and find the expression for the same. Explain why this law may be modified in practice.

(*b*) If the cost of a transmission line per mile is Rs. 10,000*a* where '*a*' is the cross-sectional area in sq. in. and if the interest and depreciation on the capital cost be 10%, estimate the most economical current density you would choose for transmission line requiring full load current for 40% of the time of

the year. Cost of generation of power is 2 annas per unit. Resistance of the conductor one mile long and one sq. in. cross-section is ·043 ohm.

As already explained, the requirements of efficiency and cheapness are opposed to a large extent. When the voltage has been decided, the size of cable for a given transmission which will give the best results can be obtained by the Kelvin's rule stated below.

The most economical size of a feeder is one for which the annual cost of energy lost in the feeder equals the annual cost of interest and depreciation on the capital cost of the conductors.

Let R = Total resistance of the feeder.

l = Total length of the feeder (positive and negative lines).

t = Time in hours of the working period during a year.

I = Average value of current during time t.

x = Cost of generation per KwH. in rupees.

Total energy lost during the year

$$=\frac{I_2Rt}{1000} \text{ units}$$

Cost of energy lost per year

$$=\frac{I^2Rt}{1000}\times x \text{ rupees}$$

Substituting $R=\frac{\rho l}{a}$

Cost of energy $=I^2\,\frac{\rho lt}{a}\times\frac{x}{1000}=\frac{P}{a}$ say

where P is a constant.

If y is the cost of conductor per unit volume in rupees.

Cost of conductors $=a\times l\times y$

If z% be the rate of interest and depreciation the annual cost of interest and depreciation

$$=aly\times\frac{z}{100} \text{ rupees}$$

$$=aQ$$

where Q is another constant.

$$\text{Total cost of transmission} = \frac{P}{a} + aQ$$

which is the minimum when $\frac{P}{a} = aQ$

or
$$a = \sqrt{\frac{P}{Q}}.$$

Limitations to the application of Kelvin's Law

Kelvin's Law must be applied with considerable caution because it does not always give a conductor size that is suitable. This is quite obvious because the cost of energy, interest and depreciation charges have no relation with the physical aspects of the problem such as resistance, voltage drop and temperature rise etc. Thus for two exactly similar systems having identical demands the cable sizes should be the same but if energy costs, interest and depreciation charges are different in the two cases the application of Kelvin's law would give entirely different cable sizes.

In selecting the size of the conductors, the cost of copper is not the only consideration, the permissible temperature rise must also be taken into account.

Again for insulated cables the cost of insulation must also be considered. This increases with the size of the cable but not in proportion to the area of cross-section.

(*b*) If total length is l miles

Cost of conductors $= 10000a \times l$

$$\therefore \quad Q = 10000 \times l \times \frac{10}{1000} = \text{Rs. } 1000\,l$$

Annual cost of energy lost

$$= \frac{I^2Rl \times a}{1000}$$

$$= \frac{I^2}{1000} \times l \times x \times \rho \frac{l}{a}$$

$$= \frac{I^2}{1000} \times 365 \times 24 \times \frac{40}{100} \times \frac{2}{16} \times l \frac{\cdot 043}{a}$$

$$P = \frac{I^2}{1000} \times 365 \times 24 \times \frac{40}{100} \times \frac{2}{16} \times \cdot 43\,l$$

Now
$$a^2=\frac{P}{Q}$$
$$=\frac{I^2}{1000}\times 365\times 24\times \cdot 4\times \frac{2}{16}\times \frac{\cdot 043\, l}{1000\, l}$$
$$\frac{I^2}{a^4}=\frac{10^6}{438\times \cdot 043}$$
$$\frac{I}{a}=\frac{1000}{\sqrt{18\cdot 8}}=231 \text{ amps. per sq. in.}$$

Example 8. Calculate the most economical size of a two core feeder supplying a consumer at a distance of 1500 yards from the sub-station, the feeder being assumed to carry constant current of 300 amps for 10 hours a day. Interest and depreciation are 10% per annum and the cost of electrical energy is $1\frac{1}{2}$ annas per unit. The capital cost of the feeder which varies with the area is Re. 1 per lb.

The resistance of 1000 yards of conductor one sq. inch cross-section is ·0243 ohm. and the weight of one cubic inch is ·32 lb.

Hours of supply per year = 3650

Resistance of feeder $=\frac{\cdot 0243\times 1500\times 2}{100\times a}=\frac{\cdot 0243\times 3}{a}$

Cost of energy lost per year $=\frac{I^2Rt}{1000}\times x$

$$=\text{Rs. } 300\times 300\times\left(\cdot 0243\times\frac{3}{a}\right)\times\left(\frac{3650}{1000}\right)\times\left(\frac{3}{32}\right)$$

$$=\text{Rs. } 900\times 9\times \cdot 0243\times\frac{365}{32\times a}$$

Weight of conductor $=a\times 1500\times 2\times 36\times \cdot 32$ lbs.

Cost of the conductor $=a\times 3000\times 36\times \cdot 32$ rupees

Interest and depreciation

$$=\frac{1}{10} \text{ of cost of conductors}$$

$$=300\times 36\times \cdot 32\times a \text{ rupees.}$$

$$300\times 36\times \cdot 32\times a=900\times 9\times \cdot 0243\times\frac{365}{32\times a}$$

$$a^2=\frac{3\times2\cdot43\times365}{32\times32\times4}=\frac{2660}{32\times32\times4}$$

$$a=\frac{51\cdot57}{64}=\cdot805 \text{ sq. in. } Ans.$$

Example 9. A 3 wire feeder is half a mile in length, the two outers having each a cross-section of 0·25 sq inch and the neutral ·125 sq. in. It carries 100 amps. in the positive outer and 80 amps. in the negative outer. The voltages across each of the two sides is 250 volts. Sp. resistance for copper is 2/3 microhm. per inch cube.

(*a*) Calculate the total power loss in the feeder.

(*b*) Determine the resistance and cross-section of a two wire feeder which will transmit 180 amps. at 250 volts with the same power loss.

Resistance of each outer $=\dfrac{\rho l}{a}$

$$=\frac{2\times880\times36}{3\times\cdot25\times10^6}=\cdot0844 \text{ ohm.}$$

Resistance of mid wire $=2\times\cdot0844=\cdot169$ ohm.

Watts loss in positive outer

$$=I_1^2R_1$$
$$=100^2\times\cdot0844=844 \text{ watts.}$$

Watts loss in negative outer

$$=80^2\times\cdot0844=540 \text{ watts.}$$

Watts loss in neutral $=20^2\times\cdot169=67$ watts.

Total loss $=1451$ watts.

If the power loss is to be the same in the two wire system then

$$I^2R_2=1451$$

$$R_2=\frac{1451}{180\times180}=\cdot0447 \text{ ohm.}$$

This is the total resistance of the two wire feeder.

Total area of cross-section in 3 wire system

$$=\cdot25+\cdot25+\cdot125=\cdot625 \text{ sq. in.}$$

In 2 wire feeder

$$R_2=\cdot0447 \text{ ohm.}$$

$$\text{Area of cross-section } a = \frac{\rho l}{R_2}$$

$$= \frac{2}{3} \times \frac{1}{10^6} \times \frac{2 \times 880 \times 36}{\cdot 0447}$$

$$= \cdot 945 \text{ sq. in.}$$

Total area of cross-section $= 2 \times \cdot 945 = 1 \cdot 89$ sq. in.

Hence for the same distance of transmission for the same power loss :

$$\frac{\text{Weight of copper for three wire transmission}}{\text{Weight of copper for two wire transmission}}$$

$$= \frac{\cdot 625}{1 \cdot 890} = \cdot 33 \text{ or } 33\%.$$

Example 10. A three wire feeder has each outer having a resistance of ·1 ohm and the neutral having a resistance of ·2 ohm. The voltage across each of the two sides is 120 V at the generators bus-bars. 50 lamps each taking a current of one amp. are connected in parallel across each side at the end of the feeder.

(*a*) (*i*) Find the voltage across each load ; (*ii*) output of each generator.

(*b*) The load between positive outer and mid-wire becomes 70 lamps and that between the negative outer and mid-wire becomes 30 lamps, find

(*i*) Current in each outer.

(*ii*) Current in the mid-wire.

(*iii*) Line drops in each outer and the neutral.

(*iv*) Voltage across each load.

(*v*) Output of each generator.

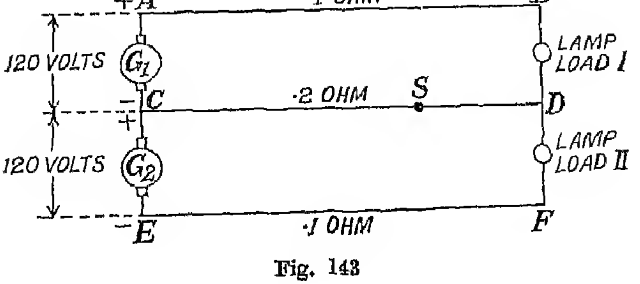


Fig. 143

(*a*) Current in positive outer $=50$ amps.

Currentin negative outer $=50$ amps.

Current in neutral wire $=50-50=0$ amp

Voltage drop in the positive outer $=50\times\cdot1=5$ volts

Voltage drop in the negative outer $=50\times\cdot1=5$ volts

(*i*) p.d. across lamp load I $=120-5=115$ volts.

p.d. across lamp load II $=120-5=115$ volts.

(*ii*) Output of each generator

G_1 or $G_2=$ voltage $\times$ current $=120\times50$

$=6000$ watts $=6$ Kw.

(*b*) (*i*) Current in positive outer $=70$ amps.

Current in negative outer $=30$ amps.

(*ii*) Current in the middle wire from D to C

$=70-30=40$ amps.

(*iii*) Voltage drop in positive outer A to B

$=70\times\cdot1=7$ volts.

Voltage drop in negative outer F to E

$=30\times\cdot1=3$ volts.

Voltage drop in mid wire from D to C

$=40\times\cdot2=8$ volts.

(*iv*) Voltage across lamp load I

$=120-7-8=105$ volts

Voltage across lamp load II $=120+8-3=125$ volts.

Note. Voltage across lamp load II is obtained by the application of Kirchoff's second law to circuit CDFE.

Suppose point E is at zero potential.

Then potential of point C is 120 V and the potential of point D is 128 V. Similarly the potential of point F is 3 volts.

$\therefore$ p.d. across points D and F $=128-3=125$ volts.

(*v*) Output of generator $G_1=$ current $\times$ volts

$=70\times120=8400$ watts $=8\cdot4$ Kw.

of generator $G_2=$ Current $\times$ Volts

$=30\times120=3600$ watts $=3\cdot6$ Kw.

Example 11. Assuming that the current taken by each lamp in example 10 remains one amp., what voltage will there be across loads BD and DF if the neutral wire broke at point S in case (*b*) with load I of 70 amps. and load II of 30 amps.

$$\text{Res. of lamp load I} \quad = \frac{\text{Voltage across load I}}{\text{Current of load I}}$$

$$R_1 = \frac{105}{70} = 1{\cdot}5 \text{ ohms.}$$

$$\text{Resistance of load II, } R_2 = \frac{125}{30} = 4{\cdot}17 \text{ ohms.}$$

When the neutral wire breaks the two loads I and II are in series and must take the same current.

The total resistance $= {\cdot}1 + {\cdot}1 + 1{\cdot}5 + 4{\cdot}17 = 5{\cdot}87$ ohms.

There is now 240 volts across this resistance and the

current $I = \dfrac{240}{5{\cdot}87} = 40{\cdot}9$ amps.

The voltage across load I $=$ current $\times R_1 = 40{\cdot}9 \times 1{\cdot}5$
$= 61{\cdot}35$ volts.

Voltage across load II $= I \times R_2 = 40{\cdot}9 \times 4{\cdot}17 = 170{\cdot}5$ volts.

It should be noted that the larger load of 70 lamps works on a considerably low voltage and the smaller load of 30 lamps is operating at a very much higher voltage than the rated value.

Example 12. A current of 80 amps is carried by a $\frac{37}{16}$ feeder run from a central station to a point $1\frac{1}{2}$ miles away. The resistance of 1000 yards length of cable is ${\cdot}214$ ohm. at the working temperature. Determine the required amount of boost for a feeder booster connected to the feeder and the booster output.

Total resistance of the feeder

$$= 2 \times \frac{3}{2} \times 1760 \times \frac{{\cdot}214}{1000} = 1{\cdot}13 \text{ ohms.}$$

Voltage drop in the feeder when carrying 80 amps.

$$= 80 \times 1{\cdot}13 = 90{\cdot}4 \text{ volts.}$$

This is the terminal p.d. of the booster so that the feeder must be boosted to the extent of $\dfrac{90{\cdot}4}{80}$

$= 1{\cdot}13$ volt per ampere of line current.

$$\text{Booster output} = \frac{90 \cdot 4 \times 80}{1000} = 7 \cdot 232 \text{ Kw.}$$

Example 13. Write a short note on the use of aluminium for electrical purposes as compared with copper. Also compare the resistance of copper and aluminium conductors each one gram in weight and one metre long. Given specific gravity of copper and aluminium respectively as 8·9 and 2·7 and specific resistance 1·69 and 2·78 microhms per cm. cube

Copper and aluminium are the two chief materials employed for transmitting power for which purpose a low specific resistance is advantageous Aluminium has only 61% of the conductivity of copper but for the same weight and length it has about twice the conductance of copper. It is softer than copper and has a tensile strength much less than that of copper. For a given conductance the carrying capacity of aluminium is more than that of copper because of its greater radiating surface. While copper has no rival in dynamo windings etc. where space is limited, aluminium is in extensive use for transmission lines where its higher resistivity is more than compensated by its low density. For the transmission lines the aluminium strands surround a central steel core for strength. Aluminium is also in common use for conductors in insulated cables nowadays even though for the same conductance the conductor is of a greater cross-section needing a greater amount of insulation. The greater amount of insulation in this case is more than offset by the lower cost of aluminium.

Cross-section of copper conductor one metre long and one gram in weight

$$= \frac{\text{Volume}}{\text{Length}} = \frac{\text{Weight}}{\text{Density} \times \text{Length}}$$

$$= \frac{1}{8 \cdot 9 \times 100} \text{ sq. cm.}$$

$$\text{Resistance} \qquad = \rho \frac{l}{a} = 1 \cdot 69 \times 100 \times 8 \cdot 9 \times 100 \times 10^{-6} \text{ ohm.}$$

$$= 1 \cdot 69 \times 10^4 \times 10^{-6} \times 8 \cdot 9$$

$$= \cdot 1504 \text{ ohm. per metre gram.}$$

Cross section of aluminium conductor one metre long and one gram in weight $=\frac{1}{2{\cdot}7\times 100}$

Resistance $=2{\cdot}78\times 100\times 2{\cdot}7\times 100\times 10^{-6}$

$=2{\cdot}78\times 10^{4}\times 10^{-6}\times 2{\cdot}7$

$={\cdot}0751$ ohm. per metre gram. *Ans.*

Example 14. Find the diameter of copper wire which will have the same resistance and length as an aluminium wire 162 mils in diameter. Sp. resistance of aluminium and copper respectively are 2·78 and 1·7 microhms per cm. cube.

If l is the length in cm. of each wire

Res. of aluminium wire

$$=R=\frac{2{\cdot}78}{10^{6}}\times\frac{l}{\frac{\pi}{4}\times{\cdot}162^{2}\times 2{\cdot}54^{2}}$$

And of copper wire $=R=\frac{1{\cdot}7}{10^{6}}\times\frac{l}{\frac{\pi}{4}\times d^{2}\times 2{\cdot}54^{2}}$

where $d=$dia. of copper wire

or $\frac{2{\cdot}78}{{\cdot}162^{2}}=\frac{1{\cdot}7}{d^{2}}$

$$d=\sqrt{\frac{1{\cdot}7\times{\cdot}162\times{\cdot}162}{2{\cdot}78}}=\sqrt{{\cdot}61}\times{\cdot}162$$

$={\cdot}126''$ *Ans.*

Example 15. From the following data compare the relative cost of using aluminium and copper conductors for transmitting a given amount of power a certain distance. The efficiency of transmission and the voltage at the consumer's end of the line should be equal in both cases.

Material	Sp. Resistance	Sp. gravity	Cost per ton.
Copper	1·7 microhm per cm. cube	8·9	Rs. 800
Aluminium	2·78 ,,	2·7	Rs. 1,500

For the same power to be transmitted with the same efficiency the I^2R loss in both cases must be equal.

The voltage at the consumer's end being equal the current I is the same in both cases and, therefore, the resistance of the conductors used in the two cases are equal.

Let R = Res. of conductors

l = Length of the conductors (lead and return).

A_c = Sectional area of each copper conductor

A_a = Sectional area of each aluminium conductor

$$R=\rho\frac{l}{A}$$

$\therefore$ $R=\dfrac{1\cdot7\times l}{A_c}$, res. of copper conductors

$R=\dfrac{2\cdot78\ l}{A_a}$, res. of aluminium ,,

$$1\cdot7A_a=2\cdot78A_c$$

or $$\frac{A_a}{A_c}=\frac{2\cdot78}{1\cdot7}=1\cdot63$$

Volume = Area of cross section × Length

For equal length of conductors

$$\frac{\text{Vol. of aluminium}}{\text{Vol. of copper}}=1\cdot63$$

$$\frac{\text{Weight of aluminium}}{\text{Weight of copper}}=\frac{\text{Vol. of aluminium}}{\text{Vol. of copper}}\times\frac{2\cdot7}{8\cdot9}$$

$$=1\cdot63\times\frac{2\cdot7}{8\cdot9}=\frac{4\cdot4}{8\cdot9}=\cdot494$$

$$\frac{\text{Cost of aluminium}}{\text{Cost of copper}}=\frac{\cdot494\times1500}{800}=\cdot926.\ \textit{Ans.}$$